

DESIGN PRINCIPLES OF A FLYWHEEL REGENERATIVE BRAKING SYSTEM
(F-RBS) FOR FORMULA SAE TYPE RACECAR AND SYSTEM TESTING ON A
VIRTUAL TEST RIG MODELED ON MSC ADAMS

BY

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To

Amma, Nanna, and Vooha

ABSTRACT

This thesis presents a flywheel based mechanical regenerative braking system (RBS) concept for a Formula SAE type race car application, to improve the performance and/or efficiency of the racecar. A mechanical system is chosen to eliminate losses related to energy conversion while capturing the rotational braking energy. The Flywheel-Regenerative Braking System (f-RBS) concept consists of a metal flywheel design of truncated cone geometry for the energy storage system (ESS) component and a V-belt CVT with a fixed gear for the transmission component of the RBS system. Racecar lap data and racecar specifications are used for designing/sizing the components. Mathematical models are developed for design, integration and operation of the f-RBS system. It was observed that a maximum of 27 % of energy requirements of the racecar can be supplied by the f-RBS.

Also, a Virtual test rig model is created using MSC ADAMS, an advanced dynamics/virtual prototyping software, in order to test the whole f-RBS system for performance, as a preliminary alternative to experimental testing. Initial testing is performed to validate the regenerative braking principle employed, to establish the actual operating limits of the virtual test rig and for an initial analysis of performance improvement by utilization of the f-RBS system. From the results, it was inferred that using the f-RBS concept can have a significant impact in recycling wasteful the braking energy and provide additional energy to the racecar.

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INTRODUCTION

1.0 INTRODUCTION

1.1.0 Preface

The Age of Industrialization, since the late 1800's, has been nothing short of remarkable. The world has vastly benefitted with the advancement of technology and knowledgebase which has brought about enormous socio-economic progress and consequently raised the standard of living of humankind beyond comprehension. Among the many resources which have played a major part in the growth, the role of fossil fuels has been undeniably quite significant. But the reserves of these 'nonrenewable' fossil fuels are rapidly declining and at the rate of growth of the humankind, the exhaustion of these resources in near future is inevitable. This has been one of the major causes of concern in the recent past. The other major concern in the present day has been the declining state of environment. The inefficient and excess usage of fossil fuels has caused the pollution of the environment, to the extent of its breakdown. As a result, over the past 40-50 years, the focus has shifted towards finding solutions which combat these issues, with the major areas of concentration being – efficient usage of energy resources (fossil fuels) in various applications like industry, transportation etc., finding alternative and/or renewable energy resources to supplement/replace the fossil fuel, better control strategies and treatment techniques to reduce pollution.[1]

Automobiles are one of the major consumers of fossil fuels and therefore: 1. One of the major sectors responsible for their depletion to the extent of near exhaustion; 2. One of the major contributors to the escalating environmental pollution levels. Due to these problems, in the recent past, the Auto Industry has come to an impasse where they are left with no alternative but to find alternate solutions to conserve natural resources. Over the years, the efforts in this direction have led to research and development in the following areas: [2, 3]

1. Improving engine efficiencies and performance – with the help of innovation in engine design and its components, improved control strategies etc.

2. Emission treatment and control technologies.
3. Vehicle design changes for reduction of thermal, aerodynamic and road losses.
4. Improvised Transmission design to reduce losses.
5. Hybrid and Alternative Energy Propulsion systems e.g. the Hybrid Electric Vehicle (HEV), the Fuel Cell Vehicle (FCV).
6. Recycling Braking energy – Storage and reuse of braking energy which is otherwise lost in the form of heat energy – technology that achieves this purpose is termed as a Regenerative Braking System (RBS).

With legislations in place for strict emission standards, the automobile industry currently has embraced the new development, by embedding one or more of the above mentioned technologies in commercial vehicles. Additionally, as a long term alternative to fossil fuels, the research in ‘Alternative Energy Vehicles’, especially the field of ‘Hybrid Propulsion with Regenerative braking’ has come to the forefront [4].

‘Hybrid Propulsion Vehicles (HPV)’ or ‘Hybrid Vehicles’ are vehicles in which two or more power sources propel the vehicle, in order to share the load and hence improve performance/improve fuel efficiency. Most of the current hybrid vehicles use an internal combustion engine (ICE) as one of the power sources and an alternative energy source (electrochemical battery, Fuel cell etc.) for the other source [5, 6]. By utilizing a hybrid powertrain, the load on the ICE reduces, as a result of which the size of the ICE can be reduced. Reduction of an ICE implies better fuel efficiency. Alternatively/additionally the ICE can be optimized to operate only in certain load conditions, which also results in improved fuel efficiency. Overall, the result is a more efficient vehicle.

As an additional tool to improving efficiency, current hybrid vehicles also employ regenerative braking to capture and utilize braking energy. Regenerative Braking Systems (RBS) capture the energy that is usually lost while braking, store that energy in an energy storage system (ESS) and the reuse it to

start or accelerate the vehicle [7]. In case of HEVs, regenerative braking is easily implemented as the electrical technology used to run the wheels is modified to also perform regenerative braking.

Designing and testing such a regenerative braking system for the application in a Formula racecar is the topic of this research. The later part of this chapter is as follows: In the next section, the motivation for the thesis is explained followed by the problem statement, after which the aims for the research and design are set and methodology used is explained. The chapter concludes with the thesis outline.

1.2.0 Motivation

Jayhawk Motorsports, University of Kansas is a student run organization that designs, manufactures, and builds two race cars within the given school year [8]. It has been participating in annual student design competition organized by SAE (Society of Automotive Engineers) – the *Formula SAE* for a little more than 20 years. The *Formula SAE* is a mainly a design, build and racing competition of mini-Indy type prototype racecars. These race cars are designed and manufactured by the students, in adherence to the guidelines/constraints set by SAE and with the intended market to be of nonprofessional weekend autocross racer. [6]

From 2011 onwards, Jayhawk Motorsports has decided to use its experience from *Formula SAE* and start participating in *Formula Hybrid* as well. *Formula Hybrid* is a competition similar to *Formula SAE* which involves designing, building and racing high performance hybrid and electric vehicles instead of conventional one. In *Formula Hybrid*, the guidelines allow the racecar to be fitted with a regenerative braking system [9]. It was observed that firstly in an autocross style racing with frequent braking events there is a potential for a substantial amount of braking energy, secondly, fitting an RBS to the vehicle would not only capture the braking energy, increase energy efficiency, but it could also provide the edge in performance as long as the gain can outweigh the loss due to the extra weight addition. This presented

an ideal opportunity for my research work – to study different types of RBS, choose, design and test a certain RBS for the racecar.

1.3.0 Problem Statement:

To devise a regenerative braking solution for Formula racecar application, design the system and test it on a virtual test bench.

1.4.0 Aims of Research

1. To lay groundwork for building a completely optimized regenerative braking system for Formula Hybrid racecar.
2. To design and test a viable model using computational simulation tools.
3. To find an optimum balance between the aims of design to design the system, but not compromise on safety.
4. To recreate the whole system in ADAMS for virtual testing.

1.5.0 Design Goals

1. High energy density and power density – Enough to capture the available braking energy and store the amount of energy needed to accelerate the vehicle.
2. High storage and transfer efficiency.
3. Conceptually and otherwise, simple and minimalistic in design.
4. Safe in operation.
5. Lightweight
6. Cost effective.
7. Easy to implement and operate.

1.6.0 Methodology

A thorough literature research was conducted in the areas of Regenerative Braking Systems (RBS) and its related aspects. Based on the literature review, mechanical, electrical and hydraulic RBS were found as potential contenders for the given application. The Flywheel or the mechanical RBS was chosen for further development and its various configurations were explored. Parallel-drive (parallel to the engine) acting on the front wheels consisting of flywheel and a continuously variable transmission (CVT) was finalized as the design choice. Based on the available racetrack data and racecar specifications, the potential and the requirements of the racecar were drawn i.e., the available braking energy, the energy needed for acceleration boost and required power was calculated. The data was used in the design and selection of individual components of the Flywheel RBS. [10]

A mathematical model demonstrating the CVT operation with the given design of flywheel was constructed. In order to validate, a testing environment was created using the software MSC ADAMS. The software MSC ADAMS is a “multi-body dynamics and motion analysis software” essentially used for studying the dynamics of mechanical systems. It can be used to “easily create and test virtual prototypes” and hence was found appropriate for the application [11]. The Flywheel system prototype was built and connected to a wheel (as a single substitute for the whole vehicle’s inertia) using ADAMS for the testing model. Various tests were performed on the model with road and no-road conditions using ADAMS Solver. The results from mathematical model were validated with the ADAMS simulation results and the model was deemed viable for further development.

1.7.0 Overview of the Thesis

In the chapter 2, an in-depth review of regenerative braking systems (RBS), flywheel based RBS and the parts of the RBS are presented. The chapter 3 deals with the design, analysis and drawing out of mathematical model while in chapter 4 the simulations techniques and testing model for this problem

using MSC ADAMS are presented. The final chapter deals with the details of the tests, its results, comparisons and discussion on implications before it finally goes on to drawing out conclusions and presenting the scope of future work.

BACKGROUND

2.0 REGENERATIVE BRAKING

2.1.0 Introduction

In order to understand the concept of a RBS and its impact on vehicle energy performance, a simple example is presented:

Consider a 300 kg (~ 661lbs) vehicle moving at an initial speed of 72 km/h(~ 45mph).Now, on braking the vehicle to a speed of 32 km/h(~ 20 mph) the amount of energy spent is around 47.8 kJ using the equation given below,

$$E_k = \frac{1}{2}mv^2 \quad (1)$$

Where, E_k : Kinetic Energy of the vehicle; m : Mass of the vehicle and v : Velocity of the vehicle

Ideally, this is the amount of energy available for capturing at each instance of braking. If regenerative braking was used on such a vehicle it would be able to capture this amount of energy and reuse this same energy which would otherwise have been lost in the form of heat, sound etc. Now, even if we suppose that the efficiency of the brake is 25% of this, there would still be an amount of 11.85 kJ (25% of 47.8kJ) of energy available at each braking instance, which shows the amount of energy that can be utilized for beneficial causes. This energy is roughly, neglecting all losses, enough to accelerate a car from 0 km/h to around 32 km/h (using equation (1)). This stored energy using RBS can be reutilized for different purposes, either to help improve performance or fuel efficiency, in either case assisting in ‘Load Sharing’.

‘Load sharing’ or ‘Load averaging’ can simply be defined as sharing of the power requirements of the vehicle between a primary and secondary propulsion/energy storage unit [12, 13]. In the recent research and systems that are currently being utilized, this load sharing, using RBS, is achieved by three functions:

1. Providing acceleration boost, in required situations (hence some of the peak power requirements are being handled by RBS).
2. Idling requirements: during idling the engine can be switched off and the energy stored in RBS can be used to restart the vehicle and the engine. [14]
3. Supplemental propulsion unit for peak/ non-peak power requirements. [15]

In this whole process, RBS also essentially functions as a brake system. But due to heavy torque demands at emergency braking situations RBS alone would not be sufficient; hence it needs to be a system supplemental to existing proven friction braking.

2.2.0 Parts and Operation of an RBS

An RBS mainly consists of two parts – the Transmission and the Energy Storage system (ESS). Depending on the arrangement of the RBS, the transmission is made to transmit energy from the wheels (or drivetrain) to/ the ESS and vice-versa. ESS can store the kinetic energy in different forms depending on its type [1]. The RBS and its types can be classified on the basis of the type of energy storage system (ESS) and the type of transmission. The choice for the transmission is usually dependent and subsequent to the type of ESS selected.

2.3.0 Types of ESS for RBS application

2.3.1.0 Electrochemical Battery:

In an electrochemical battery, the energy is stored in the form of chemical energy and released in the form of electrical energy. These have been one of the most preferred forms of energy storage systems adopted across a wide range of applications due to their compactness and cost. Batteries are also commonly used in modern automobiles to power accessories and for startup of the internal combustion engine. In recent years, in the search of alternative means of propulsion there has been extensive research and development in batteries as a good form of ESS for automobiles. This has resulted in the advent of

the 'battery electric vehicle' (BEV) and the hybrid electric vehicle(HEV), which have gained momentum in the industry [4]. These vehicles (BEVs and HEVs) use electric motor/generator pairs to propel themselves and to recapture braking energy (electric RBS) and the power source is the battery. The regenerative braking system uses a generator at the wheels or drivetrain to convert the rotational energy into electrical and store them in the battery, and when needed the electric motor utilizes the same energy to impart momentum to the vehicle.

However, the electric battery has had its demerits. One of the demerits of the electric battery is the inherent losses that accompany the energy transformations, due to which, the transfer efficiency can be quite low [16]. The other demerits are that, they have a low specific power (power per unit weight of storage system), a low storage efficiency which diminishes which each charge/discharge cycle and lack of required service life span. These reasons contribute to a limited range for battery electric vehicle (BEV) due to which, hybrid electric vehicles (HEVs) have become fairly more successful as they use an internal combustion engine, with its relatively high specific energy and power, to supplement the battery. There has been ongoing research into other electrochemical batteries like Li-ion and NiMH etc., for better performance for these applications. [4]

2.3.2.0 Ultra Capacitor Storage

Ultra capacitors are special capacitors which have the capacity to store considerable electrical energy at low voltage [4]. In a capacitor, the electrical energy is stored in the form of positive and negative charges. These unlike charges are separated and stored in two parallel plates with an insulator between them. Here the capacity to store energy is directly proportional to the area of the plates and the permittivity of the insulator/dielectric and inversely proportional to the distance between the plates. Ultra capacitors have any or all of these characteristics, for more storage. There is ongoing research in this area to find ways to make use of Ultra capacitors as well as combining both Ultra capacitors and battery in order to improve the efficiency. [4]

2.3.3.0 Hydraulic and Pneumatic Energy storage

In hydraulic and pneumatic ESS, the energy is stored by compressing a fluid and storing in an accumulator, with the accumulator being operated either mechanically or pneumatically. The energy is stored in the form of a pressurized liquid or gas. So, during regenerative braking, utilizing a hydrostatic transmission consisting of a pump–motor, the energy is captured and stored in the accumulator and the compressed fluid is then used to run the hydraulic motor to run the vehicle. This has been a well-researched technology and has been implemented in various concept vehicles, some of them currently in the market. Quite recently, as a part of the Cooperative Research and Development Agreement (CRADA), Ford Motor Company collaborated with the United States Environmental Protection Agency (USEPA) Advanced Technology Division, and built an automobile to test out Hydraulic Hybrid Vehicle technology. A hydraulic regenerative braking system called the “Hydraulic Power Assist”, which was developed by Eaton Corporation (was called “Hydraulic Launch Assist” by them), was successfully tested on this vehicle. It contained a hydro-pneumatic accumulator for storage, using hydrostatic transmission (variable displacement pump/motor) for the conversion of energy [17]. The downsides to Hydraulic/pneumatic ESS are that they are very heavy with low energy density and have excessive noise problems associated with them.

2.3.4.0 Kinetic Energy Storage System or Flywheel Energy Storage

The idea behind this concept is to capture and store the mechanical or rotational kinetic energy of the wheels in the same form, in a heavy rotating mass or the “flywheel”. This way, if a mechanical variator is used for transmission there won’t be any losses associated with the energy transformations as energy is being transmitted in mechanical form throughout. But in many cases with flywheels for energy storage, and a non-mechanical transmission, energy transformations and consequently the associated losses exist, e.g., an electrical transmission is used in the flywheel battery (FWB) designed by University of Texas at Austin, Center of ElectroMechanics (UT CEM) [13]; in such cases flywheels are used for their high

specific power, high specific energy (depending on the material of the flywheel), high tolerance to charge discharge cycles, long service life and low cost. Due to these advantages, flywheels are being used for Energy storage in various applications some of which are:

1. Uninterrupted Power Supply (UPS) systems [5, 18].
2. Grid energy systems like the Beacon Flywheel Power Storage Plant in Stephentown, New York [19].
3. Standalone/Auxiliary propulsion unit and for regenerative braking in transportation.

Flywheels store rotational kinetic energy according to the equation given below:

$$E_{k,rot} = \frac{1}{2} I \omega^2 \quad (2)$$

where, $E_{k,rot}$: rotational kinetic energy

I : the rotational inertia of the flywheel about the rotating axis

ω : the angular velocity of the flywheel

This equation implies that the maximum amount of energy that can be stored in a flywheel system can be increased with either increasing the moment of inertia (I) of the flywheel or the design maximum angular velocity (ω_{max}) of the flywheel or both. But due to safety and stability concerns associated with flywheel shattering and gyroscopic effects respectively, in automobile research applications, low speed low inertia flywheels have been preferred over high speed ones. In recent times, these issues are being resolved with the advent of new materials with high specific strength, like composite materials and design of safer high speed flywheels with high energy storage capabilities has become a possibility with. Lastly, windage losses and bearing losses are a major concern when using flywheels for energy storage. [20]

3.0 FLYWHEEL BASED REGENERATIVE BRAKING SYSTEMS

3.1.0 Introduction

A flywheel simply is a heavy rotating mass which stores kinetic or rotational energy as a function of its moment of inertia and angular velocity. Over the ages, flywheels have found an application in various kinds of machines, like the potter's wheel, punch press, steam engine etc. They are used for their ability to:[1, 2, 5]

1. Smoothen out rotations and Provide continuous energy.
2. Store large amounts of energy and release at a very high rate if and when needed.

The energy storage ability of high inertia/ high speed flywheels has been exploited to a good amount in the past. Using flywheels provides the advantage of a high power (storing and releasing both) ESS, compared to conventional batteries. Due to which, Flywheel Energy Storage (FES) systems are being used to replace electrochemical batteries in the field of power supply for Uninterrupted Power Supply (UPS) systems, grid power storage requirements and pulse power requirements. In comparison, the application of FES in the field of transportation has been more of a recent phenomenon. Safety issues with rotating steel flywheels namely disintegration and dislocation in their housing, coupled with high windage and other losses had but barred their development in the transportation sector. With the arrival of new materials with higher specific tensile strength such as Composite materials, better technology in transmission(better CVTs), bearings (e.g., magnetic bearings) and vacuum housing systems, FES for vehicular applications is being revived. [5, 21]

FES has been applied as Standalone propulsions systems (instead of traditional engines), as auxiliary propulsion units in hybrid vehicles, and as regenerative braking units in the transportation sector. Most of the systems implementing flywheel storage have regenerative braking functionality to varying degrees,

from idle stop operation to providing short burst of acceleration, to propelling the vehicle in different load conditions. [14, 15, 22, 23]

In the next section a brief history of flywheel energy storage systems in transportation is presented followed by literature review of Flywheel RBS. The chapter concludes by presenting a brief summary of transmission technology for Flywheel RBS.

3.2.0 Literature Review

One of the earliest major applications of FES in vehicles was the ‘*Electrogyro*’ bus. Around the end of 1940s, Oerlikon (Switzerland) developed the ‘Electrogyro’ bus which was a city bus with exclusively flywheel based propulsion system for short range of around 6 km. It had a huge 1500 kg steel flywheel which ran up to a top speed of 3000 rpm, was recharged at charging points by electric supply and the bus ran using a motor/generator setup. It was tested and it ran successfully first in Yverdon-les-baines, Switzerland. Due to the huge weight of the flywheel machine and the associated safety (gyroscopic, bearings etc.) and efficiency issues, although it was practically pollution free, the running of this bus was discontinued after a few years of service [1, 5, 24]. Another instance was demonstrated by the Garrett Ai Research company, when in 1960s they produced a flywheel energy storage unit with electric motor/generator unit (electrical transmission) which was tested and implemented successfully on a prototype New York R32 Subway car. In the 70s, with the beginning of the energy crisis Garrett went on to design and test the same technology for other commercial applications like transit bus, a concept train and commercial automobiles [1, 25]

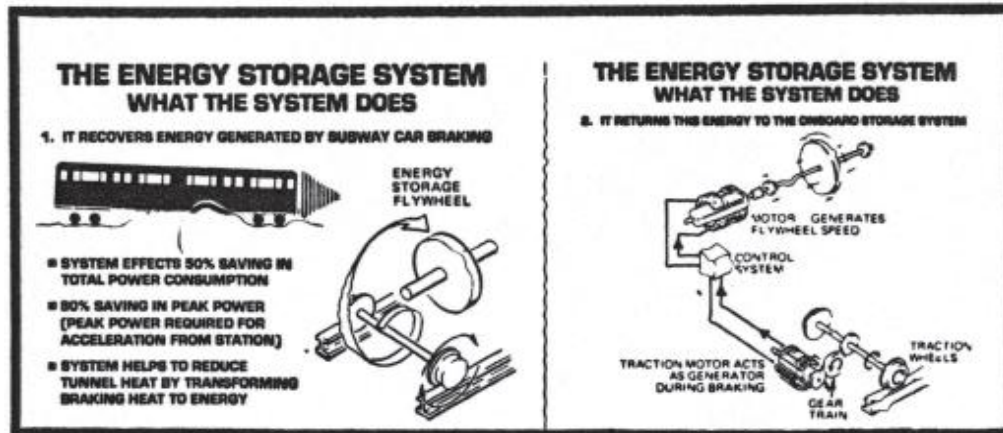


Figure 1 : Flywheel Propulsion for rapid rail cars, Garrett Ai Research Corporation(1974) [26]

About the same time, there were several Government sponsored studies on the feasibility and different configurations for Flywheel and Flywheel Hybrid propulsion for commercial automobiles [27, 28]. As a culmination of these studies, during the end of 70s, General Electric(GE) came up with its own version of the Flywheel bus or the gyro bus using a 1.5 ton flywheel and electric motor/generator for transmission [24]. Also, General Motors (GM) extensively studied the flywheel hybrid for their new concept commercial vehicle, which they designed and simulated in 1986. This system had a mechanical transmission (CVT), but after the simulation it was realized that the system (engine/flywheel hybrid) didn't match the expected requirement, hence the project was discontinued [28]. Leyland Bus in the U.K. successfully tested and demonstrated the mechanical regenerative braking system in their busses around the 80s. The system consisted of a composite flywheel (developed by British Petroleum) and the Leyland CVT for transmission. [1, 29]

During the early 1990s, Chrysler had developed the 'Patriot', a concept car for Le Mans 24 hour race. It was a gas turbine-electric hybrid vehicle with a Flywheel Regenerative Braking System used for the purpose of load leveling, with a composite flywheel. The project was dropped later due to issues in safety and packaging. With the development and usage of the composite materials for flywheels, the research in flywheels propulsion picked up immensely [1, 5, 30]. One of the most significant events in recent times, was that FIA authorized the use of Hybrid drivetrains for Formula 1 2009 season. Due to

this development, various Regenerative Braking System “Kinetic Energy Recovery System (KERS)” options including flywheel systems were explored and devised. A good example of such a flywheel KERS is the system developed by Flybrid LLP which has high speed carbon filament flywheel with Torotrak CVT [22]. This system will be described in the next section with other reviewed literature.

Note: In following sections the terms Flywheel-KERS or Flywheel-RBS may be used interchangeably to imply the same.

3.2.1.0 Schilke, N.A.(1986) [28]

As a culmination of government sponsored, GM research and other studies in the fuel economy potential of flywheel hybrid vehicles, GM designed a concept engine-flywheel hybrid drive system for a compact car and predicted its performance using analytical tools. Initially, for the powertrain concept two versions were considered, one was a system which had two mode operation and the other with one mode operation. A CVT is used for transmission in both cases. The two mode drive discontinued due to high level of complexity in design and hence the simpler one mode drive was chosen. The one mode drive was a parallel drive system in which wheels or accessories (of the vehicle) were driven either by the engine or the flywheel and similarly, the energy recovered by regenerative braking charged the flywheel or the accessories. There was no method of charging the flywheel directly through the engine. All the flywheel energy was captured only through braking. The design of flywheel was based on the GM Cymbal Flywheel and had a maximum rotational speed of 12000 RPM, and energy storage of 60Wh. After laboratory tests and computer simulation, the performance was predicted to achieve a significant gain in urban fuel economy but the highway fuel economy could not meet the required efficiency levels and hence the project was discontinued.

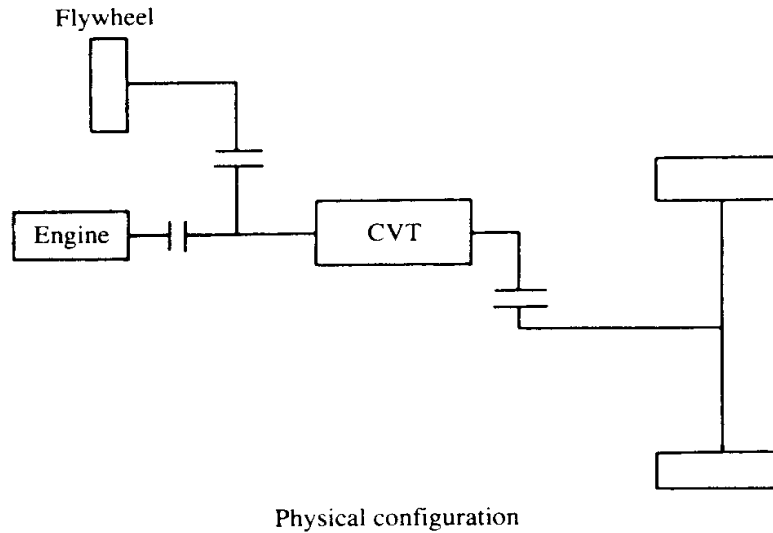


Figure 2: One Mode configuration of the Engine-Flywheel Hybrid Drive by GM [28]

3.2.2.0 Jefferson and Ackerman (1996) [23]

The paper describes a flywheel based energy storage system designed as a standalone propulsion unit or the main propulsion unit in a hybrid setup, for a railcar application. The system comprised of a steel flywheel and a mechanical variator (i.e., CVT) for matching the speeds of output drivetrain and the flywheel and also in order to affect power transfer from/to the flywheel by the variation of the CVT ratio. The Flywheel of flywheel energy storage (FES) could be charged by an onboard or external power source (battery, engine etc.) en-route or at stops, and by regenerative braking and the charged FES was used to propel the railcar. The system was tested on a laboratory test rig and successfully demonstrated on a minitram. The test rig setup used a composite flywheel, a KOPP CVT and an induction motor as power source (simulating vehicle). Open and closed loop tests were used to understand CVT control and gauge system performance. The minitram system used a steel flywheel of 4 MJ storage capacity. The results from the Minitram system showed that on using ‘flywheel only’ propulsion, the energy savings were around 24%. It was concluded that having a hybrid powertrain with a flywheel-variator system and a compact constant power source like an Internal Combustion Engine (ICE) or Fuel Cell would result in a fuel efficient vehicle of large range.

3.2.3.0 ETH -Hybrid III : Dietrich et al (1999) [15]

The parallel hybrid drivetrain design of ETH – III hybrid vehicle project has been described. The design was successfully tested and implemented. An ICE, an electric motor, a flywheel, a CVT with a wide gear ratio and a battery (5 kWh capacity) are all part of the drivetrain. Three friction clutches control the power flow between the output and the sources. The drivetrain has the capability to operate in different drive modes, due to which the vehicle can be powered individually by the engine, flywheel or the motor, or the motor and flywheel can operate in a combined mode. The control of the drive modes was one the basis of the output load requirement. Flywheel could be charged by the engine or by regenerative braking, but the regenerative braking potential was limited. The entire powertrain system was tested on a dynamic test bench after the completion of individual component testing and then the system was integrated into a Multi-Utility Vehicle and tested on a chassis-dynamometer test bench.

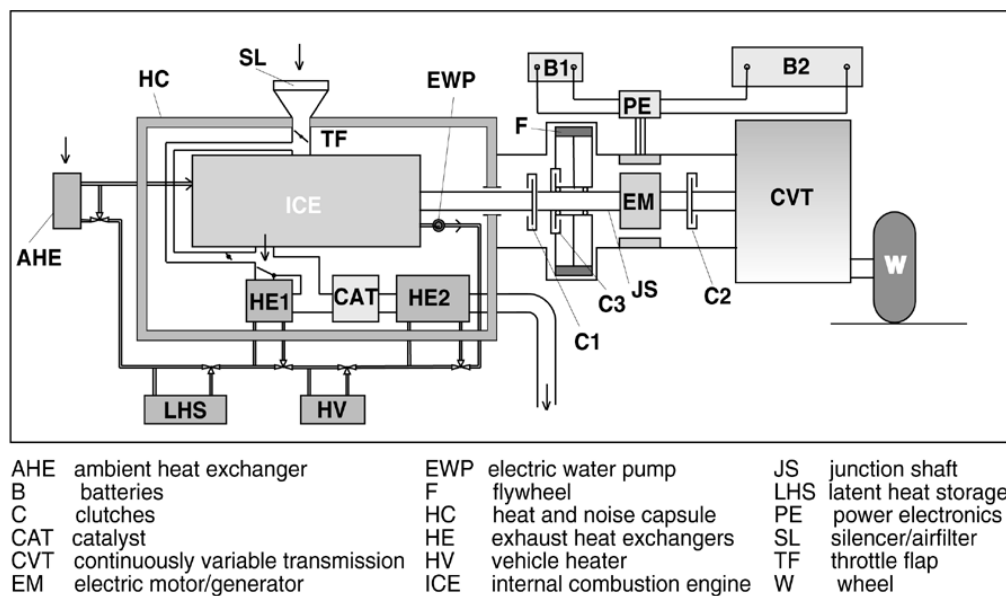


Figure 3: ETH-Hybrid III Powertrain [15]

3.2.4.0 Hayes et al (1999) [13]

This paper describes the University of Texas-Center for ElectroMechanics (UT-CEM)'s Flywheel Battery (FWB) project for a hybrid electric transit bus. A Flywheel Battery (FWB) is common

nomenclature for systems in which the transmission is an electric motor/generator but the energy is stored in rotational form in a flywheel. So when the FWB is used for regeneration, the rotational energy of the wheels is converted into electrical energy and back into rotational energy at the flywheel using an electrical motor/generator and vice versa for acceleration. UT-CEM developed computer models to simulate hybrid electric drivetrain with FWB to calculate the design requirement. The FWB used in this demonstration had a 2 kWh composite flywheel and a high speed permanent magnet motor/generator with magnetic bearings. The FWB was of “partially integrated topology” which means that the flywheel and motor/generator are separate components but ride on the same shaft and in a single housing. Initial testing was carried out using a titanium flywheel and the thermal analysis results obtained were included to optimize the design. All components are individually bench tested for safety and optimization. The paper describes the system in pre-vehicle integration testing phase.

3.2.5.0 Zero Inertia Powertrain at Eindhoven University (2001-06)[2, 14, 31]

The EcoDrive project and related work done at the Eindhoven University of Technology presents an innovative powertrain concept called “the Zero Inertia powertrain (ZI)” with the aim of improve fuel efficiency of vehicle by efficient engine utilization methods and hybrid strategies to achieve load sharing. Optimal engine operation is ensured by the use of the CVT and additionally the flywheel power assist helps in load sharing. The Planetary Gear System (PGS) through its three degrees of freedom connects the primary shaft of Metal push belt CVT (engine side), the secondary shaft of the CVT and the flywheel. One application of the powertrain, called the “IdleStop and Go” (SG) powertrain, for use in start-stop operation of a vehicle, is presented. The system operates such that when the vehicle comes to a stop or to idle, the engine is shut off. When the vehicle needs to resume motion, the flywheel launches the vehicle as well restarts the engine. This way the fuel consumed during idling in conventional systems is saved. The flywheel is charged by the engine as well as through regenerative braking although that functionality is limited.

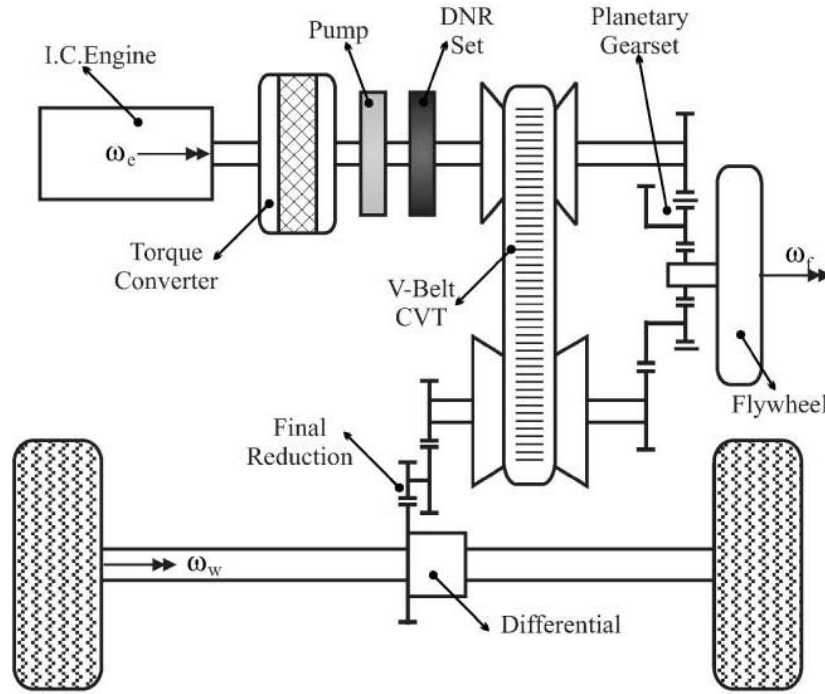


Figure 4: ZI Powertrain Layout [31]

3.2.6.0 Ayala et al (2008)

The authors propose, design and test a mechanical energy storage system using a flywheel, to be used for regenerative braking purposes in vehicles. The design is compact to the extent possible so as to minimally affect drivetrain changes while integration. It consists of a carbon fiber flywheel (moment of inertia = 0.11 kg-m^2) for energy storage and a two stage planetary gear system (PGS) for power flow control and transmission. The system operates in parallel with the conventional engine. In the PGS, the sun gear of the first stage is connected to the planet carrier of the second. The flywheel is connected to the second stage sun gear and the wheel to the first stage planet carrier. Both the systems have a common ring gear, and a frictional brake is used on the ring gear to control the power flow. By braking the ring gear energy flow is either from flywheel to wheel or otherwise, depending on their initial velocities.

The system operates in three modes – Neutral, regenerative braking and Flywheel assisted acceleration. Two powertrain versions were proposed one consisting of the PGS and the other had an

additionally CVT for increased range of utilization after the ring gear has been stopped. A computational model was formulated for design and its results compared with those obtained experimentally on a scaled physical model tested in laboratory using a dynamometer for vehicle behavior, which validated the operation of the design and the model. Furthermore, computational simulations of a full car model including the Mechanical energy storage system were conducted using different standard drive cycles to confirm better fuel efficiency and reduced emissions.

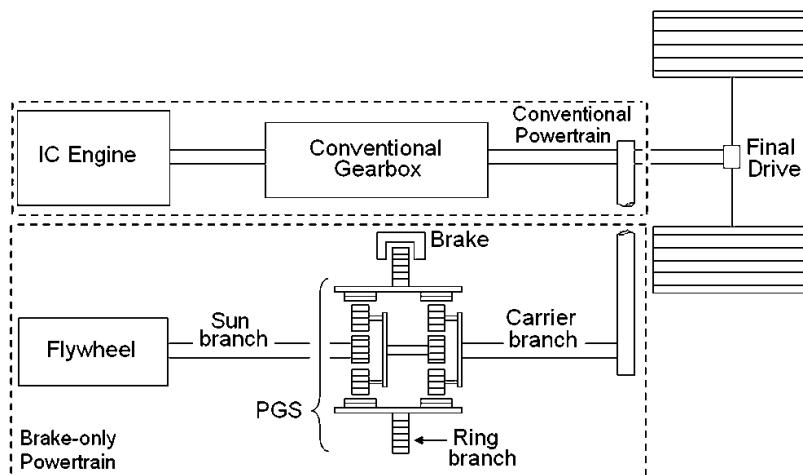


Figure 5: Mechanical Energy Storage System - Brake Only Version [32]

3.2.7.0 Cross and Brockbank (2009)

The paper presents a mechanical Kinetic Energy Recovery System (KERS) developed by Flybrid Systems LLP using a Torotrak Continuously Variable Transmission (CVT), mainly for Formula 1 (F1) application and also for motorsport and commercial automobile applications. The aim of the project was to come up with a KERS solution for F1 application (in response to FIA's recent regulation in F1 to allow hybrid drivetrains with KERS for performance boost and fuel efficiency) and also make it commercially viable. In keeping up with requirements, compactness, high safety and cost effective were some of goals set for the design. As a result, the mechanical KERS system developed used a high speed carbon filament flywheel as the ESS and a Torotrak toroidal traction drive CVT, and the power rating and max speed is given as 60 kW and 64500 RPM respectively. The power flow control is managed by an electrohydraulic

control system which controls the clutches which activate/deactivate the KERS and also the CVT control. The system and its individual components are tested for safety and optimized for performance in the laboratory. The complete system is tested on test bench using a dynamometer and a V-8 engine and the computer simulations of the system are performed on an US FTP 75 drive cycle. The results establish that using the mechanical KERS can contribute to 21% of the energy requirements of a vehicle.



Figure 6: Flybrid KERS using Torotrak CVT [33]

3.3.0 Transmission or Variator

The energy stored in a flywheel is given by the equation (2), and so to recover or add energy from/to the flywheel the inertia I or the angular velocity ω needs to be varied. In other words, the flywheel exerts a torque only if its angular momentum is varied and vice versa for torque on the flywheel, in any case the angular momentum i.e., the product of moment of inertia (I) times the angular velocity(ω) needs to be varied, implying either or both of I and ω need to be varied. Hence in case of flywheel energy storage the transmission needs to be a variator which can vary either of those quantities.

A continuously variable transmission (CVT) is one of the most common forms of variator mechanisms, which varies the ω . Any CVT should have a wide ratio range to accommodate and match both the speeds at each end (flywheel and wheel), and needs to continuously variable across the range to initiate energy transfer. For example if a flywheel storing energy is running at 10,000 RPM and the vehicle wheel it is connected to through the CVT is running at 1000 RPM, then before initiating energy

transfer between both of them, the CVT needs to match up speeds by switching to required gear ratio (Mechanical: gears, variable diameter pulley, toroidal surfaces; Electrical: motor/generator, hydrostatic: pump/motor etc.). Once both the sources (flywheel and the wheel) are connected and at equilibrium, the ratio needs to be varied i.e., in this scenario from ratio = 10:1 to 5:1 or 20:1, continuously to vary the speeds so as to initiate an energy transfer or a momentum exchange. Based on the literature, the types of CVT commonly used are described below:

3.3.1.0 Hydrostatic CVT:

A Hydrostatic CVT consists of a variable displacement pump and hydraulic motor combination which control the ratio and the variation of it. Such systems have the inherent disadvantage of energy conversion losses as energy is being converted from rotational to hydraulic back to rotational. The advantages of such systems are that they are easily available, easily controllable, durable, and have good power density. On the other hand their disadvantages are that they have poor efficiencies, high on weight addition high noise level. [1, 5]

3.3.2.0 Electrical or Electromagnetic CVT:

These are systems consisting of electrical motor/generator pairs as seen in UT-CEM Flywheel battery [13] or any other Flywheel battery project [21, 34, 35]. An electrical CVT is shown below in the figure. M1 and M2 are two motor/generator pairs connected to each of the sources- flywheel and wheel side shaft. For energy transfer from flywheel to wheel, the rotational energy is converted to electrical by generator of the M1 pair decelerating the flywheel and this electrical energy is used to run the motor of the M2 pair. Energy is transferred in a similar manner from wheel to flywheel during regenerative braking.[2]

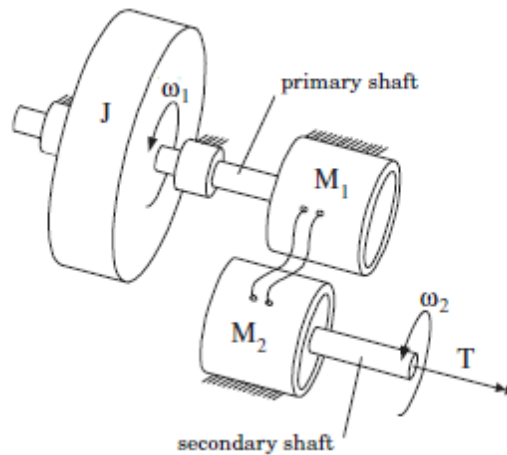


Figure 7: Electrical CVT [2]

These systems inherently have energy conversion losses. Other than that they are most easily controllable, are compact, can have a wide and infinitely variable ratio, are durable and have very low noise levels. The downsides are that these systems are expensive and the transfer efficiencies tend to be less (due to conversion losses).

3.3.3.0 Mechanical CVT:

Mechanical CVTs can be of various types. Rolling traction drives like toroidal CVT and pulley belt type are some of the common types. Planetary Gear System combinations are also for the same purpose[32]. With mechanical systems there are no energy conversion losses hence the transfer efficiencies can be quite high. That also means the power density is also high. They are simple in design, economical, compact and have low noise levels. They are not very durable and need maintenance due to constant wear and tear. [1, 12]. The design and operation of the common types of CVT is explained below:

Toroidal CVT System: Typical toroidal systems consist of discs with toroidal surfaces and rollers which sit between the discs. The rollers can be tilted. The input and output shaft are connected to the discs on either side of the rollers, so that with each angular position of the roller, a certain speed ratio is set

between the input and output shafts. Hence, the variable ratio of the drive is controlled by tilting the rollers which subsequently can be controlled with the help of an actuator. This design of toroidal systems provides for a continuously varying gear ratio and wide ratio range. [22] Toroidal CVT systems have the benefit of being compact, easy to implement and control, having low noise levels, and with efficiencies greater than 90%. [12]

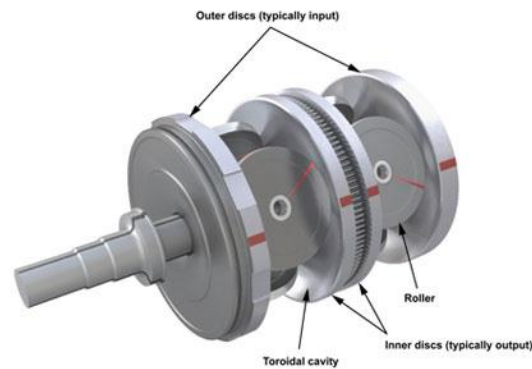


Figure 8: A Torotrak Toroidal CVT[36]

Variable Diameter Pulley CVT: A common Pulley-Belt CVT consists of two V-belt pulleys connected through a belt sitting in them. One or both of the pulley sheaves of each pulley can be moved along the axis of rotation. By movement of the sheaves the effective diameter of the running belt on each pulley can be varied due to which the overall gear ratio changes. The design of the system is such that the effective diameter changes, due to sheave movement are complimentary to each other i.e., if the diameter of the first pulley increases then the second pulley diameter will decrease and vice-versa, this in order to maintain the belt tension, as the belt length as well the center-distance between the pulleys doesn't vary. The V-belt can be made up of various materials depending on the load to be transmitted, with metal-V-Belt used for automotive gearboxes like the Nissan Xtronic CVT[37] and Rubber V-belts for ATV and Go Kart applications. Pulley Belt-CVTs are easy to control, easily available and simple in design with efficiencies on the higher side depending on the material and design. [5]

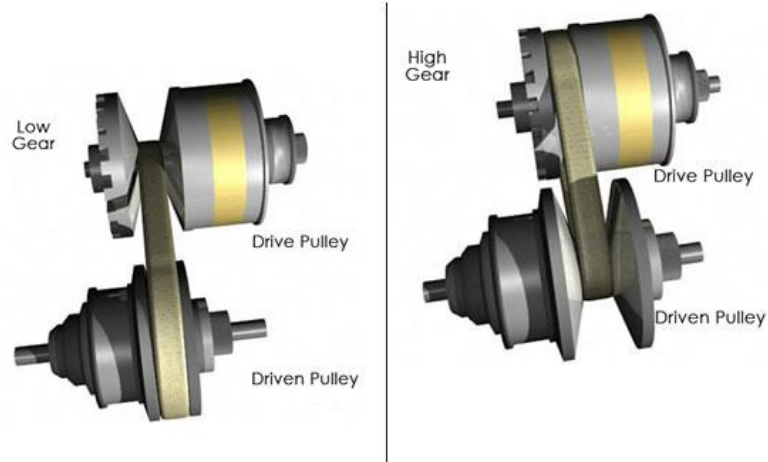


Figure 9 : Pulley Based CVT [38]

Other than the above mentioned CVT, a split power CVT can also be used using a variety of configurations, e.g., a pulley belt CVT with a PGS. In split power systems, the majority of the mechanical power is transmitted directly via fixed ratio and a part is transmitted via a variable ratio system. This way, the efficiency of power transmission increases as minor part is transmitted in a less efficient manner (variable ratio system). The variable ratio system can be any of the above mentioned systems. [1, 32]

DESIGN AND DEVELOPMENT

4.0 INTRODUCTION TO THE CONCEPT AND PRE-DESIGN

As introduced before, the aim of this research is to design and test an RBS (Regenerative Braking System) solution for a Formula SAE racecar application. Such a system will benefit the racecar in two ways:

1. Save the wasteful braking energy and reuse that energy as an additional source to power the vehicle, albeit temporarily, thus saving engine's power and consequently reducing net fuel consumption.
2. The current design is for front-wheel application in a rear-wheel drive car. So, using the RBS system to power the front wheels, provides a temporary acceleration boost to the car. This is because in a rear wheel drive car, the max engine torque deliverable to the rear wheels is traction limited. So, one alternative for extra power or torque to the car for additional acceleration is by powering the front wheels too (similar to AWD/4WD vehicles).

Before designing such a system, the design requirements need to be established. This is in order to set the design constraints for the system. Though, the aims of the design have been previously mentioned, but for the sake of convenience, the same are mentioned again below:

1. High energy density and power density – Enough to capture the available braking energy and store the amount of energy needed to accelerate the vehicle.
2. High storage and transfer efficiency.
3. Conceptually and otherwise, simple and minimalistic in design.
4. Safe in operation.
5. Lightweight
6. Cost effective.
7. Easy to implement and operate.

The next sections cover the design and the development of the RBS.

4.1.0 The RBS Concept or Flywheel RBS Concept or Concept Definition

As a first step, a base template of the system needs to be set before proceeding with the design process. On studying the various types of ESS, it was realized that for all their merits and demerits any of the ESS could be used for the application. The electrical ESS has been extensively researched and is an excellent choice due to its high specific energy, compactness and operational/implementation simplicity. But it was found lacking in storage/recharging efficiency and the transmission losses associated with energy conversion from mechanical energy to electrical energy. So, on an exploratory basis, a flywheel with a mechanical transmission was selected with the aim of overcoming these particular shortcomings of an Electrical RBS system.

With a flywheel ESS, high specific power can be achieved and depending on the design, high specific energy can be obtained as well. Additionally, flywheels have excellent recharge efficiencies and very long cycle lives. On coupling this with a mechanical transmission the conversion losses are eliminated as the mechanical braking energy is transmitted and stored in the same form. The other advantages being the system can be cost effective (depending on the design) and simple to recreate. The problems associated with flywheel systems are high weight addition and safety issues. Therefore most of the design goals are met with a Flywheel based Mechanical System.

The Flywheel RBS (f-RBS) concept consists of a low/high speed flywheel with a mechanical transmission and this arrangement is to be directly connected to the front wheel axle/spindle.

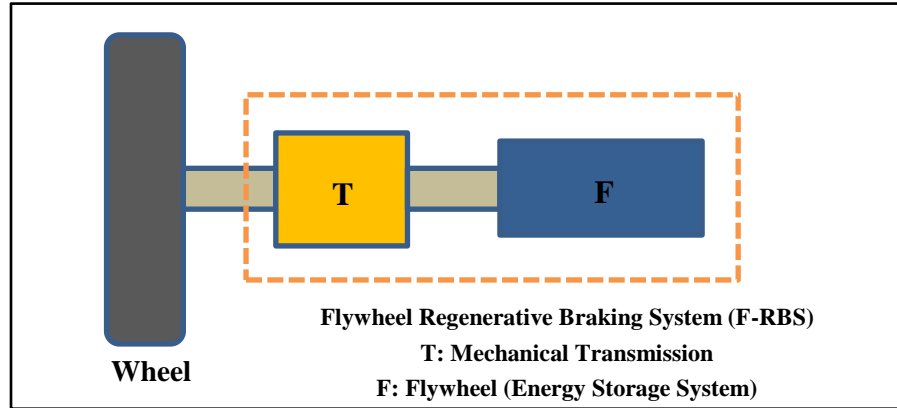


Figure 10: Flywheel-RBS Concept

4.2.0 Mechanical Transmission

After considering the mechanical variator options described previously, a variable diameter pulley CVT (VDP) was selected for the application. A rubber belt VDP has the advantage of being simple in construction and inexpensive since it is commonly used among ATVs (All-Terrain Vehicle), Go-karts and Pocket Bikes. Although in comparison with the other mechanical variator options (like the Toroidal CVT, CVP [39], the Rubber Belt –VDP is less efficient, and it still has adequate power and efficiency ratings for the given application.

4.3.0 Overall Configuration

The overall construction of the system is presented. The Flywheel RBS consists of Low Speed Flywheel connected to a flywheel fixed gearing which is connected to the primary pulley of Rubber Belt VDP CVT through a clutch arrangement. The Secondary pulley is connected to the wheel through another clutch. The purpose of the flywheel fixed gearing is to provide an overall wide gear ratio range between the flywheel and wheel. The maximum gear ratio calculated between wheel and flywheel in rough

calculations was deemed beyond the ratio range of any available rubber belt CVTs in the market. Hence, the fixed gear between the flywheel (flywheel gear) and the CVT will provide for a multiplier to the ratio of the CVT according to the equation given below.

$$n_{overall} = n_{CVT} \times n_{gear} \quad (3)$$

where, $n_{overall}$: Overall gear ratio at the wheel including that of CVT and flywheel

n_{cvt} : Gear ratio of the CVT

n_{gear} : Gear ratio of the fixed flywheel gearing

Note: In the coming sections, the design calculations for the f-RBS are presented, and the f-RBS is designed with the aim that the total inertial load of the racecar will be distributed equally between two such systems operating on both the front wheels.

4.4.0 Theory of Operation

The Flywheel RBS would operate in three modes: *Regen-Braking*, *Accel-Boost* and *Neutral*. The three modes are described below[15]:

1. *Regen-Braking*: In this mode, the vehicle energy is captured in the flywheel and the vehicle is slowed down in the process. First, the CVT ratio is controlled so as to set the overall ratio to match the ratio between the wheel and the flywheel in their original states. After matching the speeds with the help of the CVT, both the clutches are activated to connect both the wheel and flywheel to the CVT. After the connection is established, the CVT ratio is changed continuously from the original high gear ratio to lower ratios so as to initiate the required energy flow from the wheel to the Flywheel. If the need for more braking arises beyond the limits of the CVT, the braking will be switched immediately to the default brakes of the racecar.
2. *Accel-Boost*: In this mode the stored energy of the flywheel is used for accelerating the vehicle temporarily. As in the previous mode, first the CVT ratio is set so as to match the speeds of wheel

and the flywheel. Then, the clutches are activated to establish connection and the CVT ratio is switched continuously from low to high so as to initiate an energy transfer from the flywheel to the wheel. Once the flywheel energy is reduced to a certain limit or the acceleration need ends, the clutches are deactivated to disconnect the system on both ends.

3. Neutral: In the neutral mode, the system is disengaged with both the flywheel and the wheel through the clutches. The flywheel and the wheel rotate at their own speeds without any connection. This can be the case during either when there is no need for *Accel-Boost* or f-RBS assisted braking or when automatic disengagement when the flywheel to wheel speed ratio is less or more than the extremes of the f-RBS's transmission ratio.

The control of actuator mechanism in the CVT and the clutches control the operation of the system and its modes.

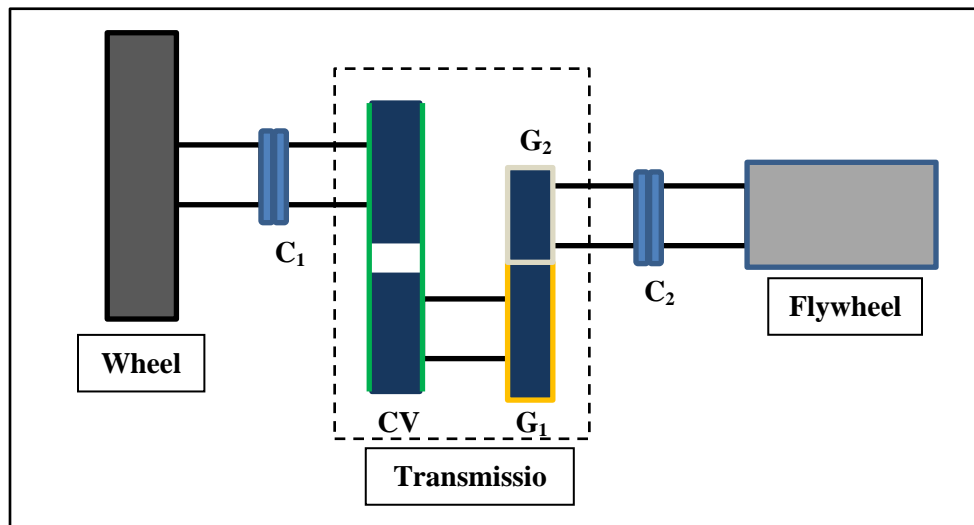


Figure 11: Overall Configuration
C: Clutch; G: Gear

The next sections present pre-design calculations and design principles of the individual components which are:

1. Flywheel
2. CVT and Fixed Gear

4.5.0 Energy Available for Braking

Before designing the RBS system the required power rating and the energy rating of the system i.e., the operating power and maximum amount of energy it can store needs to be defined. For these numbers the amount of braking energy available and the power of braking during a test lap is calculated. The data used to calculate is the DAQ (Data Acquisition) data from JMS 08' (Jayhawk Motorsports) racecar test runs. The data was analyzed with the help of **AiM Race Studio 2**© [40] Analysis mode

The velocity [km/h] vs. time [s] plot is presented below of a full lap from the DAQ data. The plot for the lap is not a zero to zero plot.

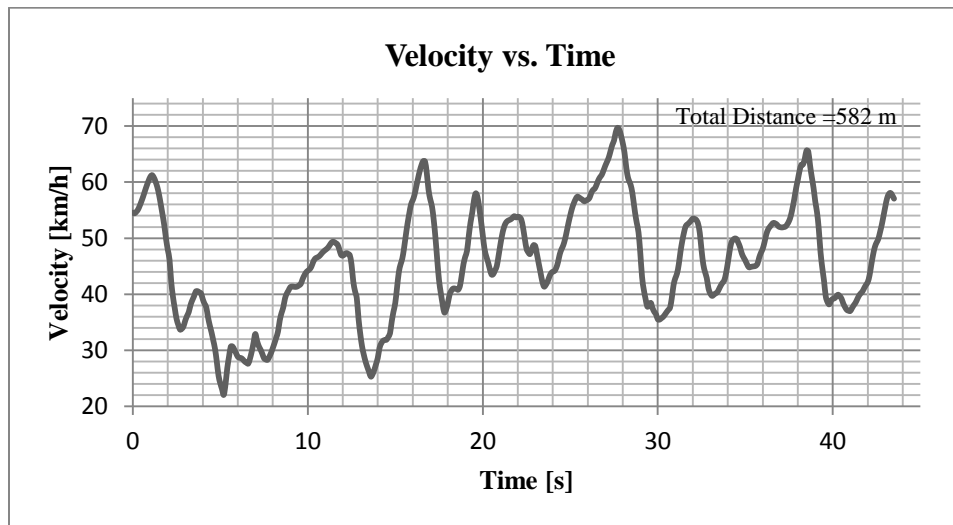


Figure 12: Velocity [km/h] vs. Time [s] Graph from DAQ Data

The kinetic energy vs. time below is generated from the above plot using the kinetic energy equation (1),

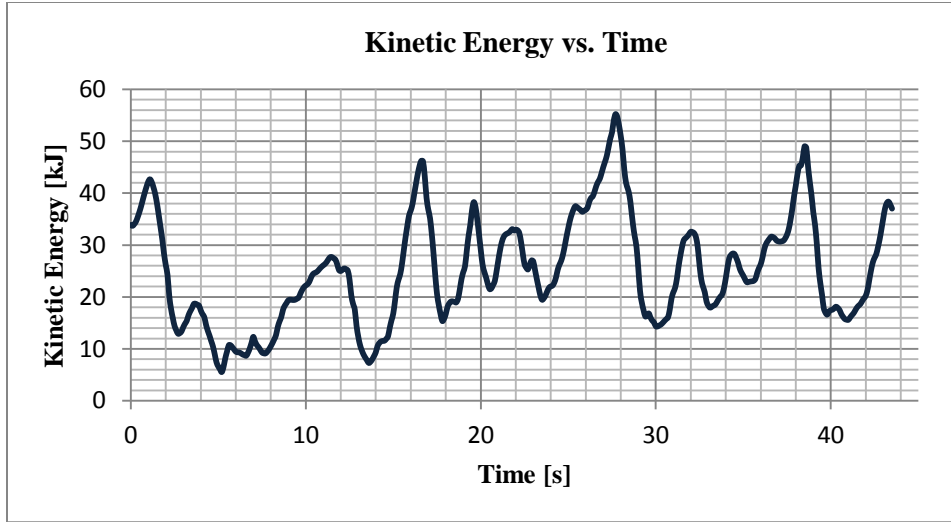


Figure 13: Translational Kinetic Energy [kJ] vs. Time [s] of the vehicle

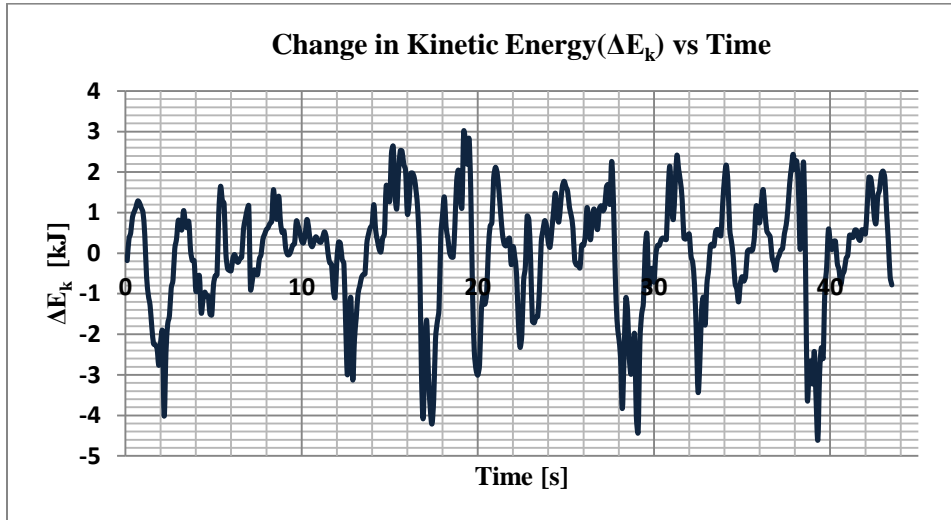


Figure 14: Change in Kinetic Energy [kJ] vs. Time [s]

In the above plot, the troughs below zero line indicate deceleration due to mostly braking. In order to verify the braking instances correctly, the brake pressure [psi] vs. time [s] plot was referred and the troughs below zero in the above plot have been compared to the brake pressure crests in the respective plot. Nine braking instances were identified with the time change for every instance, the total change in kinetic energy (ΔE_k) during every braking instance, and average braking power at each instance were calculated.

The actual available braking energy is the change in kinetic energy minus the losses (rolling, bearing, aerodynamic etc.). The losses were calculated and a plot of change in kinetic energy (including losses is presented below. In the plot, as mentioned before, the curve below the zero line indicates the braking energy below the zero line and acceleration energy above the line. So the total energy needed to accelerate the vehicle is the integral of ‘change in kinetic energy’ curve (red curve in the plot below) above the line over the whole lap plus the losses and this is the blue curve above the zero line in the plot below. In a similar manner, the total energy available while braking is the integral of the ‘change in kinetic energy’ curve below the zero line minus the losses and this again is the blue curve below the zero line in the plot below.[22]

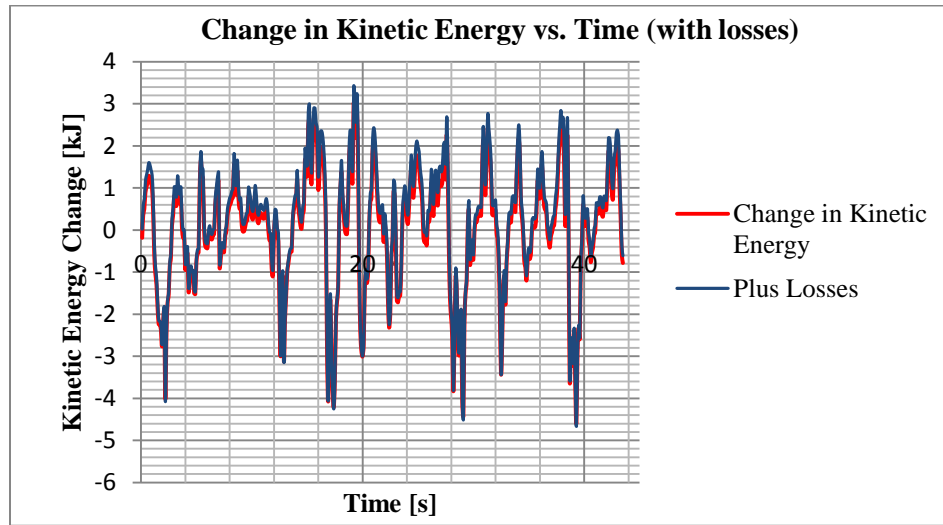


Figure 15: Change in Kinetic Energy (With Losses) vs. Time

Hence, the actual available braking for any braking instance is given by the change in translational kinetic energy of the vehicle plus the change in rotational kinetic energy of the whole vehicle (including that of wheels, half shafts etc.) minus the losses due to aerodynamic drag and rolling resistance. Bearing losses are neglected at this point. And for the total energy for the lap, it is the summation of the individual parts over the entire lap.

$$\Delta E_{a,b} = \Delta E_k + \Delta E_{k,rot} - W_{losses} \quad (4)$$

$$E_{a,b,lap} = \sum \Delta E_{a,b} = \sum \Delta E_k + \sum \Delta E_{k,rot} - \sum W_{losses} \quad (5)$$

$$W_{losses} = F_{losses} \times d \quad (6)$$

$$F_{losses} = F_d + \mu_r(mg) \quad (7)$$

Where, $\Delta E_{a,b}$: Available braking energy at any instance of braking

ΔE_k : Change in translational kinetic energy of the vehicle during any braking instance

$\Delta E_{k,rot}$: Change in rotational kinetic energy due to all rotating parts

W_{losses} : Work done to overcome aerodynamic and rolling resistance losses

F_{losses} : Force at vehicle/wheel due to losses (aero and rolling)

d : Braking distance

F_d : Aerodynamic Drag Force

μ_r : Coefficient of rolling friction [41]

g : Acceleration due to gravity = 9.8 m/s²

In the equations the bearing losses were neglected. Hence, from the data and plots the total braking energy available for the whole lap was calculated by summation of the values at individual braking instances. In a similar manner the total acceleration energy needed in a lap was calculated which was change in kinetic energy plus the losses in this case. The values obtained are presented in the table 1.

Using this data, it can be inferred that the f-RBS system to be designed needs to store the maximum available braking energy per wheel , i.e., 9.61 kJ \approx **10kJ** and needs to operate at the maximum braking power i.e., 6.43 kW \approx **7 kW**. Also a point to note from the table is that, if the f-RBS system is used to power the vehicle, assuming an 80% efficiency of the transmission system, in a single lap 75 kJ of the 278 kJ needed can be supplied by RBS. This means 27% of the vehicle power needs are being supplied by the RBS.

Table 1: Available Brake Energy from Lap Data

Calculated property	Total	At the front axle (assuming 50% bias)	per wheel (Total/2)
Maximum Braking energy in any instance [kJ]	-38.434	-19.217	-9.609
Average braking energy (per Braking Instance) [kJ]	-21.039	-10.520	-5.260
Maximum braking power in any instance [kW]	-25.722	-12.861	-6.430
Average braking power [kW]	-16.057	-8.028	-4.014
Total Brake Energy available per lap [kJ]	-189.351	-94.676	-47.338
Total Acceleration energy needed per lap [kJ]	278.200	-	-

Table 2: Race Car Specs

RACE CAR SPECIFICATION (JMS 09')	
Mass of the vehicle (including the driver) (m)	650 lb. or 294.835 kg
Wheelbase (L)	65 in
Height of the center of gravity from ground (h)	10 in
Total equivalent moment of inertia of rotating components(including wheels) (I_{eq})	1.111 kg-m²
Rolling resistance coefficient (μ_r)	0.015 [41]
Aero Drag force total (@ 45 mph from Car Wing Data)	106 N
Drive wheels	Rear wheel drive

4.6.0 Mass Addition vs. Performance

For a racecar, addition of mass implies invaluable reduction of performance, so in the design of the f-RBS the mass needs be optimized so that the loss in performance due to the additional mass of the f-RBS

system is completely surpassed by the gain in performance due to the power addition accompanying the f-RBS. Hence an analysis was performed to calculate how the mass addition affects the performance of the racecar. As a measure for performance, the minimum time taken to travel 75 m. starting from rest (t_{75}) is used. Using the calculations and a Matlab code created by the Formula SAE team (Jayhawk Motorsports), to this purpose, the following plot is generated.

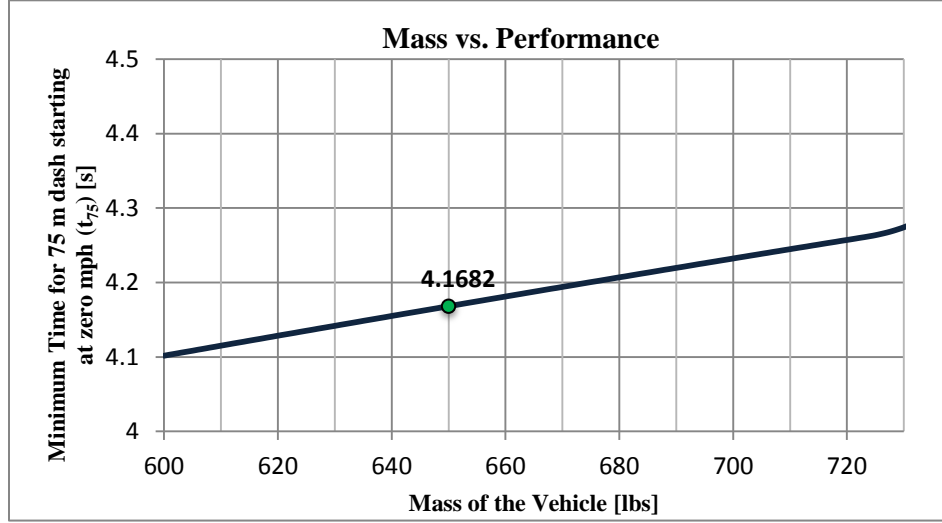


Figure 16: Effect of mass addition in the performance of the vehicle

This plot shows how the t_{75} varies with the addition of mass. The default point in green shows the time for the vehicle mass of 650 lbs. which is the assumed base mass of the vehicle with driver and without the addition of the f-RBS system. This curve doesn't include the improvement in performance due to the f-RBS power. The Matlab code calculated the t_{75} with the help of the Torque curve of the engine, also considering the gear shift RPM etc. The Matlab code was developed based on the following equations [41]:

$$F_x = \frac{\tau_e N_{tf}}{r} \quad (8)$$

$$F_{lim.traction} = \mu_{s,lim} W_{rear} \quad (9)$$

$$W_{rear} = W_{r,s} + W \frac{h}{L} + F_{aero,down} \quad (10)$$

$$a_x = \frac{F_x (or F_{lim.traction}) - F_{losses}}{m} \quad (11)$$

where, F_x	: Tractive force at the wheels obtained from the engine
τ_e	: Engine Torque
N_{tf}	: Combined ratio of transmission and final drive
r	: radius of the wheels
$F_{lim.traction}$: maximum or limiting traction due to limiting coefficient of static friction
$\mu_{s,lim}$: Limiting or peak coefficient of static friction
W	: Weight of the vehicle
W_{rear}	: Weight on the drive wheels (rear)
$W_{r,s}$: Static Weight distribution- rear
$F_{aero,down}$: Aerodynamic Downforce

Based on Figure 16), around 70-80 lbs. seems to be a good mass constraint for the flywheel system as vehicle time t_{75} doesn't get affected drastically with the addition of 80-90 lbs. The t_{75} goes from 4.1682s at default mass of vehicle at 650 lbs. to 4.3215s at 740 lbs.

5.0 CVT DESIGN/SELECTION

For the current application, the CVT/Variator design needs to operate in a particular manner:

1. The CVT continuous ratio change should be externally controllable
2. The rated power rating should match that required by the f-RBS
3. It should have a wide ratio so as to affect size reduction of flywheel

The reasoning for the above points – as per kinetic energy equation of the flywheel – equation (2), the amount of energy stored in a flywheel is directly proportional to the Moment of inertia I and the square of the rotational velocity ω . So, either a flywheel can be designed to be of less mass and operate at higher velocities or vice versa. For the given application and due to limit set by available maximum range of CVT (explained later) and safety concerns, a requirement of lower operating speeds of flywheel was decided upon. This automatically implied that the flywheel design needs to compensate on moment of inertia and hence the mass of the flywheel. As mentioned, CVT design/selection sets a limit on the design of the flywheel. Since the available energy is a function of moment of inertia, the maximum flywheel speed pertaining to the maximum energy available varies. The CVT range is the constraining factor to the maximum speed flywheel can achieve and hence to the maximum energy it can capture out of the available energy. In order to maximize capturing either the mass/moment of inertia needs to be increased or the range needs to increase.

Mechanical Variators/CVTs, presently used in the industry, are of different design and operational capabilities due to the varying needs of the applications. They are known by different nomenclature depending on the application. One of the types of CVTs are the ones used in the automotive sector, the ‘Torque converters’ or ‘Belt-CVTs’- these CVT’s used in the scooters, tractors etc. are designed either for high performance or for high fuel economy. The common belt CVTs used in scooters or ATVs are designed to *automatically* shift from the low to high ratio and vice versa depending upon the output torque load or speed load with the help of springs, weights and other torque/speed sensing equipment. In

case of heavy duty industrial application (machines utilizing electric motors etc.,) the CVTs better known as ‘Variable Speed Drives’, are designed to vary output speed(driven shaft) continuously for a constant speed of electric motor(driver shaft), either manually controlled or with automatic operation on the basis of load. [42, 43]

The need of the present system is as follows: as mentioned before in the system operation, at the need of *regen-braking* or *Accel-boost* mode initiation, the CVT ratio needs to be set so as to synchronize the speeds of the driver and driven i.e., the wheel and flywheel or vice versa. And once they are engaged through the clutches, the gear ratio needs to be switched continuously from the current value to either a higher or lower gear, so as to initiate an energy transfer. Hence, for this operation the CVT needs to be externally controllable as per need.

After examining all the variator/CVT/Variable speed drive options available, it was decided that either of the options given below are suitable for the present application:

1. A Variable speed drive of ‘Fixed Center type Drive’ design e.g., Speed Selector Fixed center variable pulley drive – series 45 which has a manually controllable pulley. [42]
2. A modified design of torque converter/CVT system used in ATVs and scooters e.g., a Comet Symmetric 40 series Torque converter.[44]

In case of the Variable Speed Drives, the advantages are that these systems are available in the power rating required for the f-RBS system and also that they can be externally controlled. But, on the downside, they are heavy in construction and are designed for a system in which the driver (usually an electric motor) operates at constant RPM and driven varies, whereas in the context of the f-RBS in this thesis both the driver and driven operate at varying RPM. This fact needs to be examined while selecting a variable speed drive.

A modified design of a CVT/Torque converter is desirable, as they are simple in construction, lighter weight and available in the power rating required.

5.1.0 Modified Torque Converter Design

A typical torque converter and clutch system or a CVT system used in ATVs and scooters is of similar design as to the diagram shown above. It is a Variable diameter pulley (VDP)-type CVT using rubber (Kevlar reinforced or polyester reinforced) V-belt. The operation of the original system is as follows: Initially the belt sits at the lowest on the left pulley (say driver pulley) and at the highest point on the right pulley (say driven pulley) to give a lower gear ratio. On the back of one driver pulley plates are roller or spring weights, which with increase of rotational speed push the pulley plate into the other driver pulley plate (due to centrifugal forces) and hence the belt moves up on the driver pulley. When the belt position changes on the driver pulley, the belt position on the other (driven pulley adjust itself), due to the tension maintained in the belt with the help of spring at both pulleys. And in the process pushes the driven pulley plates away from each other. So, with increase of rotational speed at the driver the belt position moves from a lower to a higher on the driver and vice versa on the driven pulley, and consequently the gear ratio changes from lower to a higher gear continuously, smoothly and automatically. [44]

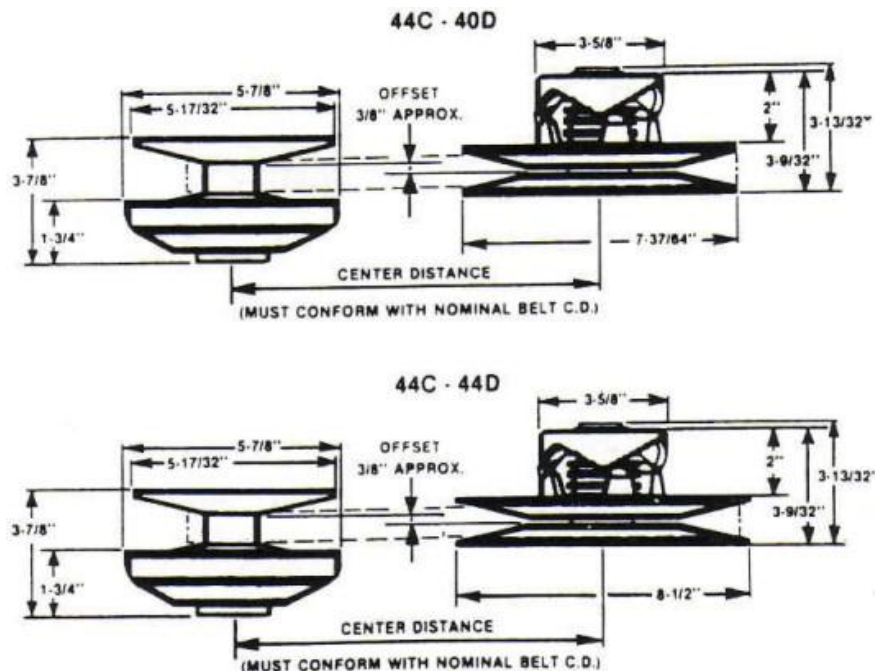


Figure 17: Example of a 'Torque Converter-Clutch' type CVT used in ATVs- Comet System [44]

This whole process needs to be externally controlled and not be driven automatically by the roller/spring weights. In the modified design the roller/spring weights mechanism needs to be taken out and replaced with some kind of an actuation mechanism, like hydraulic or electronic actuators at the pulleys. The actuator will control the pulley plate's position and hence control the gear ratio. With the pushing together or pulling apart of the pulley plates at any one pulley, the other pulley plates at the other pulley also vary accordingly due to the effect of springs present in the system or both the pulleys are actuated which adjust the position and maintain tension in the belt. This way it will operate in the same manner as a controllable variable speed drive, and gear ratio of the variator can be controlled as per the requirement with the help of a control system.

5.2.0 CVT Ratio Analysis

To understand the extremities of gear ratio values while braking and accelerating using an f-RBS system, a plot of Vehicle rotational speed in RPM versus gear ratio for different flywheel speeds is shown.

In the plot, the wheel angular velocity in RPM is on the y-axis and for different flywheel angular velocities the required gear ratio is calculated and is on the x-axis. The gear ratio is given by,

$$n = \frac{N_{Flywheel}}{N_{wheel}} \quad (12)$$

where, n : gear ratio

$N_{flywheel}$: Angular velocity of flywheel in RPM

N_{wheel} : Angular velocity of vehicle wheel in RPM

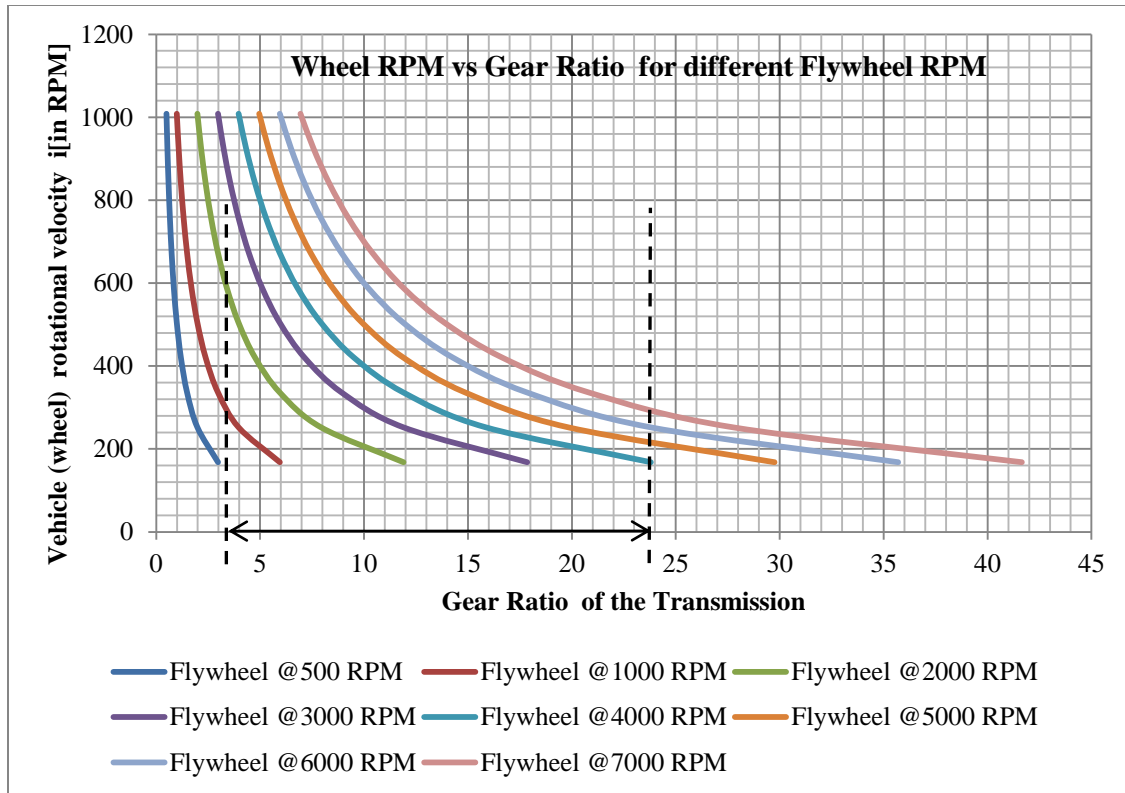


Figure 18: Plot to determine the range of vehicle and flywheel operating speeds for a given range of transmission

The wheel angular velocity is calculated by assuming the radius of the wheel as 0.254 m (= 10 in) using which, for example, it can be calculated that the angular velocity of 1008 RPM corresponds to a vehicle translational speed of 60 mph. It was assumed that the f-RBS wouldn't be utilized beyond 60 MPH of vehicle speed.

From the plot, it can be seen that if the vehicle is travelling say at 60 mph (i.e., wheel angular velocity of 1008 RPM) and if the flywheel was rotating at 500 RPM then, in order for a smooth connection between the wheel and f-RBS, the gear ratio of the f-RBS transmission (of the f-RBS) needs to be set to 0.5 to 1, as in the case of f-RBS braking. Similarly, during f-RBS assisted acceleration, if the vehicle were travelling at 10 mph (i.e., wheel angular velocity of 168.0676 RPM) and the flywheel rotating at 7000 RPM, then for a smooth connection the gear ratio needs to be set at 41.65 to 1.

It can be seen that, in order for the f-RBS system to operate at its full potential, either the transmission needs to have a wide range or the flywheel maximum speed is lower, or both. After exploring various CVT options in the industry and studying the above plot, it has been decided that a gear ratio range for the f-RBS transmission of 4 -24 i.e., 4 to 1 being the highest gear and 24 to 1 being the lowest, would be optimum and achievable. One way to obtain the ratio range is by using a combination of a CVT with ratio range of 1 – 6 (i.e., 1:1 to 6:1) and fixed flywheel gearing of 4:1.

Also from the plot, it can be inferred that for a 4 – 24 ratio range, the maximum wheel angular velocity at which f-RBS can be used for braking is 800 RPM (i.e., translational velocity \approx 48 mph) and a minimum angular velocity of 336 RPM (i.e., a translational velocity \approx 48 mph) if a f-RBS assisted acceleration were to be used with flywheel rotating at 7000 RPM. This way, the plot can be used to determine the range of operation of the f-RBS for the vehicle, which in this case is 3000-7000 RPM for the flywheel and around 300-800 RPM (i.e., 20-48 mph range). The plot is also used to set the maximum angular velocity design constraint for the flywheel which is 7000 RPM.

6.0 FLYWHEEL

The flywheel is the energy storage component of the regenerative braking system. In applications with frequent power requirements, flywheel as an ESS is an attractive alternative, with negligible losses in cycle and storage efficiency over time.

According to the equation of the kinetic energy stored in a rotating flywheel, given by equation (2), flywheel designs are classified on the basis of angular velocities of operation and moment of inertia characteristics. On this basis, flywheel designs can be Low speed or High speed with Low or high moment of inertia. With the needs of the application, low speed or high speed flywheels are employed. Both the low speed as well as high speed flywheel design with different geometries can be utilized for the same stored energy requirement. Common materials like metals are usually employed for making low speed flywheels as they are heavy and are economical , although they do tend to have safety concerns of shattering at high speeds,. As for high speed flywheels, composite materials tend to be used, as they have high specific tensile strengths, consequently higher specific energies and also better failure characteristics at higher speeds.

$$t_m = \frac{\sigma_{max}}{\rho} \quad (13)$$

where, t_m : Specific strength or specific tensile strength

σ_{max} : Ultimate tensile strength

ρ : Mass density of the material

Specific energy is the amount of energy that can be stored per unit mass and is a function of the material's specific strength. So, on the basis of flywheel geometry and material there can be various alternatives to satisfy a certain design need. [45, 46]

6.1.0 Design Basics

From the kinetic energy equation, it can be inferred that in order to maximize energy storage of a certain flywheel, either the moment of inertia (I) or the operating angular velocity (ω) needs to be increased. Here, the moment of inertia of any geometry is given by

$$I = \int r^2 dm \quad (14)$$

where, r is the radius of any differential mass of the body – dm from the axis of rotation.

Now, if solid disc geometry of mass - M and radius - R is considered for the flywheel, then its moment of inertia (I_{sd}) is given by,

$$I_{sd} = \frac{1}{2}MR^2 \quad (15)$$

and its kinetic energy (E_k) equation becomes,

$$E_K = \frac{1}{4}MR^2\omega^2 \quad (16)$$

or, with the mass density (ρ) and peripheral velocity (V), the equation becomes

$$E_K = \frac{1}{4}\rho VR^2\omega^2 \quad (17)$$

For a rotating disc with inner radius (r_i) and outer radius (r_o), shattering or failing due to the centrifugal forces is a main cause of concern. The tangential and radial stress profile as a function of the radial distance from the axis of rotation (r), is given by [46]

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2) \quad (18)$$

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2) \quad (19)$$

where, σ_t : Tangential stress at any point at a radius r from the axis of rotation

σ_r : Radial Stress at any point at a radius from the axis of rotation

ν : Poisson's ratio of the material

Using the equations above, if a stress versus radial distance curve is plotted for each of stress type, the radial stress values are found to be less than tangential stress all along the radius. For any ω of flywheel the tangential stress peaks at $r = r_i$, so if at a certain ω , σ_t at r_i is greater than ultimate tensile stress for the material then the flywheel will fail and this would give $\omega = \omega_{max}$ - the maximum angular velocity of rotating disc at or after which the disc may fail. And, ω_{max} will determine maximum energy that can be stored without failing ($E_{K,max}$) for the flywheel with a given I . [45]

$$E_{K,max} \propto \omega_{max} \quad (20)$$

And,

$$\omega_{max} \propto \sigma_{max} \quad (21)$$

For a solid disc of radius R the relation between ω_{max} and σ_{max} is given by,

$$\sigma_{max} = \rho R^2 \omega_{max}^2 \frac{3+\nu}{8} \quad (22)$$

Hence, the equation (17) becomes,

$$E_{K,max} = \frac{1}{4} V \frac{\sigma_{max}}{(3+\nu)/8} \quad (23)$$

or,

$$E_{K,max} = k V \sigma_{max} \quad (24)$$

where k called the shape factor is,

$$k = \frac{8}{4(3 + \vartheta)} = 0.606 \text{ (for } \vartheta = 0.3) \quad (25)$$

Equation (24) is a general equation for maximum kinetic energy for any flywheel geometry [1, 21, 45]. From Equation (24), the specific energy (e_M) and the energy density (e_V) are obtained as,

$$e_M = \frac{E_{K,max}}{M} = k \frac{\sigma_{max}}{\rho} \quad (26)$$



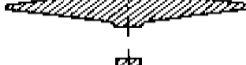



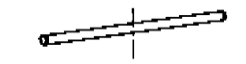
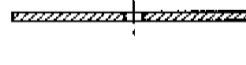

and,

$$e_V = \frac{E_{K,max}}{V} = k \sigma_{max} \quad (27)$$

It can be seen that the specific energy is directly dependent on the specific tensile strength.

Different geometries have been studied in the literature with the idea of maximizing specific energy or energy density. In this process, the shape factors for different geometries have been determined. These are presented in the table below.

Table 3: Shape Factors for different flywheel geometries[1]

<i>Flywheel Geometry</i>	<i>Cross-Section or Pictorial View</i>	<i>Shape Factor K</i>	
<i>Constant-Stress Disc (OD $\rightarrow \infty$)</i>		1.000	} <i>Suitable for Homogeneous Materials Only</i>
<i>Modified Constant-Stress Disc (Typical)</i>		.931	
<i>Truncated Conical Disc (Typical)</i>		.806	
<i>Flat Unpierced Disc</i>		.606	
<i>Thin Rim (ID/OD $\rightarrow 1.0$)</i>		.500	} <i>Suitable for Homogeneous or Filamentary Materials</i>
<i>Shaped Bar (OD $\rightarrow \infty$)</i>		.500	
<i>Rim with Web (Typical)</i>		.400	
<i>Single Filament Bar</i>		.333	
<i>Flat Pierced Disc</i>		.305	

6.2.0 Design of the Flywheel:

The design constraints for the flywheel for the f-RBS system are the following:

Table 4: List of design constraints for the flywheel

Constraint Type	Description
Energy Storage	≈10-12 kJ
Mass	< 80 lbs. (≈36.29 kg)
Space Constraint	Compact
Transmission Constraint	Gear ratio range = 4-24

Out of the above constraints, the critical constraint is the transmission subsystem gear ratio range. Due to limitation placed by the transmission ratio range, the flywheel rotating speeds above 7000 RPM cannot be utilized. Hence, a low speed flywheel design is chosen, which takes the transmission constraint into consideration and is also economical to implement.

Now, using the kinetic energy equation, considering the flywheel stores maximum available energy (≈10 kJ) at maximum utilizable rotational speed (≈ 7000 RPM), the moment of inertia of the flywheel (I_f) needs to be

$$I_f = \frac{2E_k}{\omega_f^2} = 0.037 \text{ kg-m}^2 \quad (28)$$

6.3.0 Material Selection

The specific tensile strengths for different metals are calculated to determine the material with better specific energy capability. The material physical property data is obtained using *SolidWorks*.

Table 5: Specific strength for different materials

Material	Tensile strength (σ_{\max}) [N/m²]	Density (ρ) [kg/m³]	Specific Strength (σ_{\max}/ρ)
201 Annealed Stainless Steel SS	685000000	7860	87.150
AISI Steel 1045	625000000	7850	79.618
Alloy Steel	723825600	7700	94.003
Ductile Iron	861695000	7100	121.365

From the above table, Ductile Iron is chosen for flywheel material due to its high specific strength.

6.4.0 Geometry

On the basis of Table (3), the ‘truncated conical disc’ type geometry is chosen because of its high specific strength.

The final design parameters are as given below:

Table 6: Final design parameters

Design Parameter Type	Value
Moment of Inertia (I_f)	$\approx 0.037 \text{ kg-m}^2$
Shape	Truncated Conical Disc
Material	Ductile Iron
Space limitations	Radius < 0.15 m ; Thickness < 0.15 m

Using the design parameters, a design study was performed using a CAD Software – *SolidWorks*, with the final goal of minimizing mass. *SolidWorks* was used for purpose of Computer Aided Design (CAD) and Analysis in this thesis, due to its standard, easy-to-use interface and its solid modeling and FEA (Finite Element Analysis) Capabilities. The resultant flywheel designed using *SolidWorks* CAD module, has the following dimensions and properties:

Table 7: Flywheel design (FW) dimensions and properties

Flywheel Dimension/Property type	Value of the Dimension/Property
Total Thickness (t_o)	5 cm
Inner Thickness (t_i)	1.3 cm
Outer Radius (r_o)	11 cm
Inner radius (r_i)	3 cm
Moment of Inertia (I_f)	0.038 kg-m ²
Mass (m_f)	\approx 8 kg

For future reference, this flywheel design will be referred to as ‘Flywheel (FW)’ for the sake of convenience and for avoiding confusion.

6.5.0 Structural Analysis of the flywheel:

In order to understand the safety/failure characteristics of the Flywheel (FW) while rotating at high speeds of 7000-8000 RPM, and undergoing extreme torques while charging and discharging, a structural analysis is done on the Flywheel (FW).

As an initial analysis, the ω_{max} - maximum angular velocity at which the maximum tangential stress equals the σ_{max} – is calculated for the Flywheel (FW) using Equation (18), by approximating the Flywheel (FW) geometry to a solid disc of outer radius 11 cm and a thickness of 5 cm.

Table 8: Parameters used for structural analysis and results of analysis

Input/output Parameter	Value
Radius (R_o) [m]	0.11
Inner radius (R_i) [m]	0.015
Thickness [m]	0.05
Material	Ductile Iron
Design type	Solid Disc (assumed for the sake of calculation)
ω_{max} [rad/s]	10934.53 (\approx 104417.1 RPM)
including a Factor of Safety(FOS)=2	52208.55 RPM

Max. Energy stored [kJ]	12
ω at Max. Energy[RPM]	5178.364

The flywheel design was analyzed for structural failure using Finite Element Analysis (FEA). The FEA module of *SolidWorks* - '*SolidWorks Simulation*' was used to perform this analysis. The structure was tested under the loading conditions of centrifugal forces due to rotation at **10,000 RPM** and angular acceleration of **500 rad/s²**. The stress, strain, displacement and FOS plots are respectively shown below.

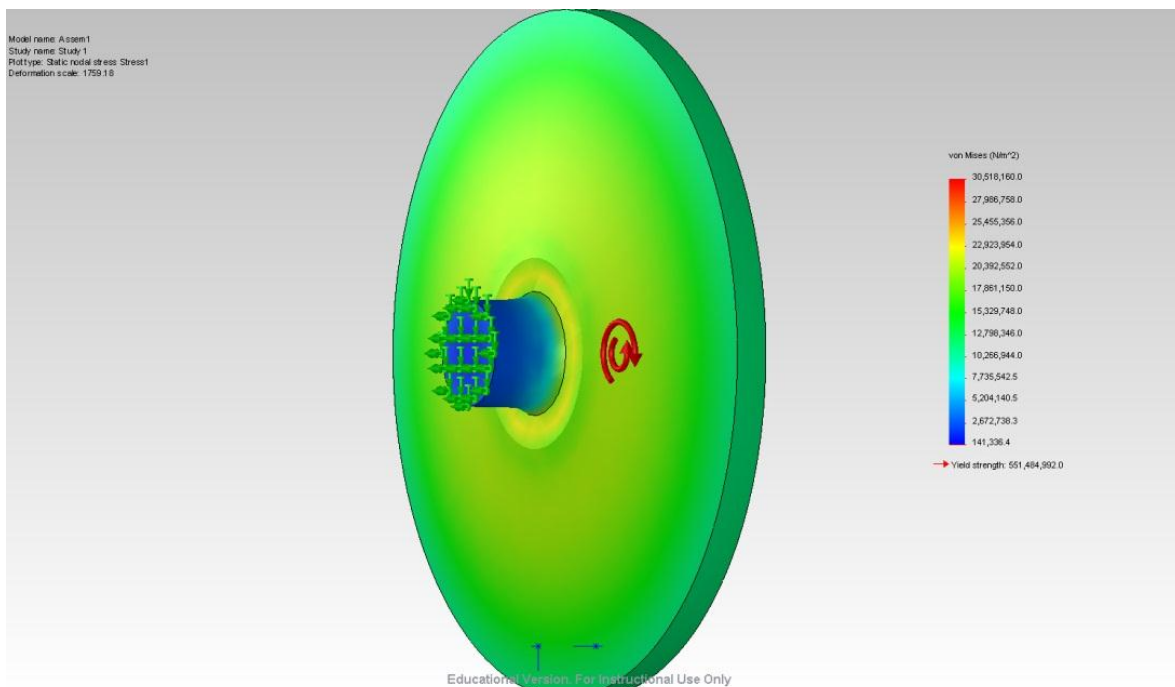


Figure 19: von Mises Stress Plot of the flywheel design (FW) using Solidworks 'Simulation' FEA module

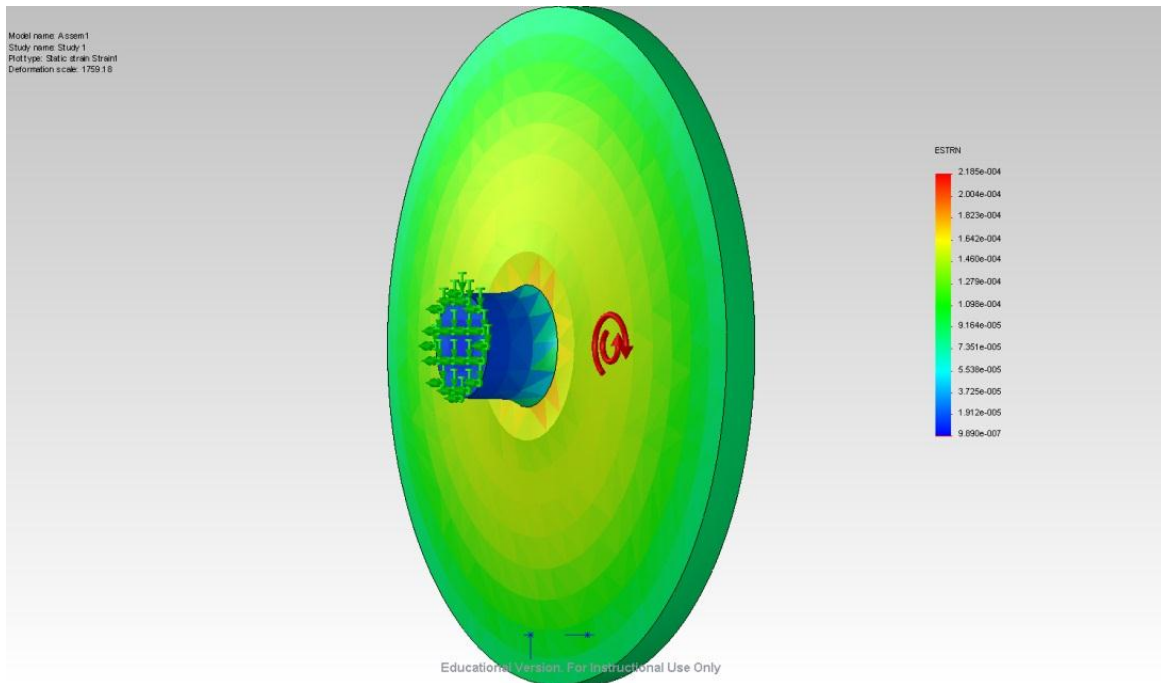


Figure 20: Strain Plot for the flywheel design (FW) using *Solidworks*' *Simulation*' FEA module

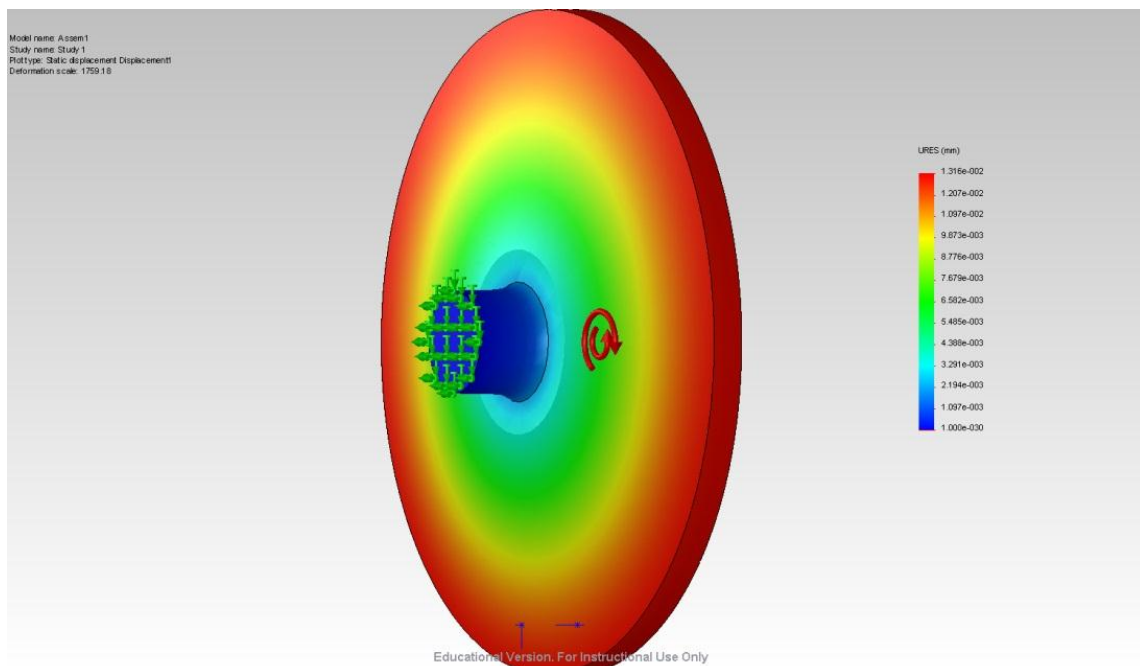


Figure 21: Deformation [mm] plot of the design (FW) using *Solidworks*' *Simulation*' FEA module

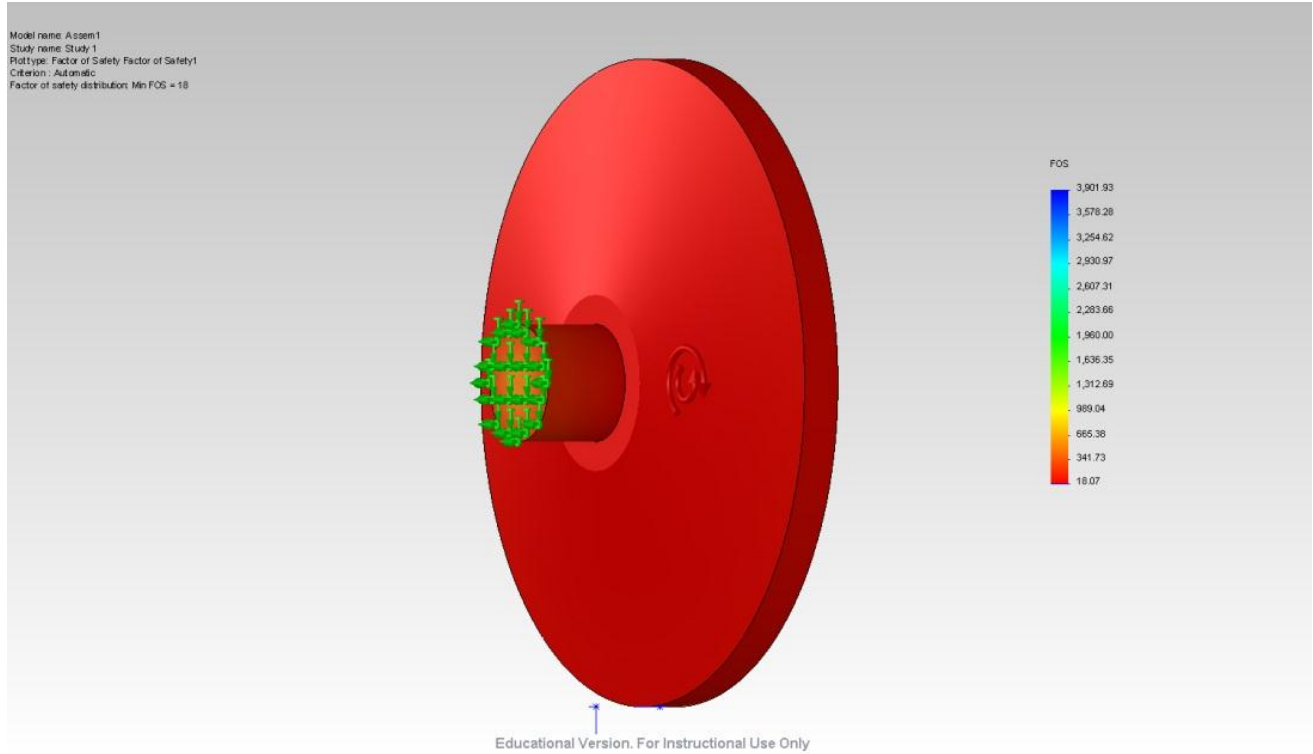


Figure 22: Factor of safety plot for the design (FW) using *Solidworks* 'Simulation' FEA module

The FEA results showed that the design (FW) under the given conditions had a minimum factor of safety of 18 and hence this design of f-RBS was finalized.

6.6.0 Mathematical Modeling of Vehicle with flywheel-Regenerative Braking System (f-RBS)

In this section, a basic mathematical model of the operation of a vehicle with the f-RBS is presented. The energy exchange between the wheel and the flywheel through the CVT/Transmission is modeled. In the derivation, the vehicle is modeled as a wheel with an equivalent mass and moment of inertia of the whole vehicle (including its rotating parts like all four wheels, driveshaft etc.).

Consider a rotating flywheel with a moment of inertia (I_f), connected to a rotating wheel through a CVT,

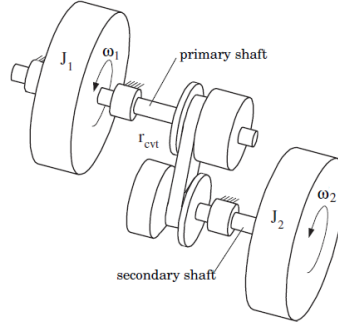


Figure 23: Flywheel + CVT + Wheel [2]

$$E_f(t) + E_v(t) = C \text{ (Constant)} \quad (29)$$

$$\frac{1}{2}I_f\omega_f(t)^2 + \frac{1}{2}I_v\omega_v(t)^2 = C \quad (30)$$

$$I_v = mR_w^2 + I_{eq} \quad (31)$$

$$N(t) = \frac{\omega_f(t)}{\omega_v(t)} \quad (32)$$

$$\frac{1}{2}\omega_v(t)^2 \times (N(t)^2 I_f + I_v) = C \quad (33)$$

$$\omega_v(t) = \sqrt{\frac{2C}{(N(t)^2 I_f + I_v)}} \quad (34)$$

Where, $E_f(t)$: Kinetic energy of the flywheel as a function of time t

$E_v(t)$: Kinetic energy of the vehicle as a function of time t

m : Mass of the vehicle

$\omega_v(t)$: Angular velocity of the vehicle/wheel as a function of time

$\omega_f(t)$: Angular velocity of the flywheel as a function of time

R_w : Radius of the vehicle wheel = 0.254 m

$N(t)$: Gear ratio as a function of time

I_v : Equivalent moment of inertia of the whole vehicle

Using Equation (34), the amount of energy transfer and the direction of energy transfer when the gear ratio between a rotating flywheel and wheel is forced to change, can be determined. For example, if the racecar described before, were travelling at 45 mph, and the flywheel of the f-RBS it is connected to is rotating at 3025 RPM, (gear ratio (N) = 4:1) then the amount of energy transfer if the gear ratio was switched from 4:1 to a higher or lower gear is determined by the Equation (34) and presented as a plot below. From the plot it can be seen that if the ratio were to be switched to 10:1 then vehicle wheel speed will reduce to 39.51 mph and flywheel speed will increase to 6641 RPM, and hence energy is transferring from the wheel to the flywheel i.e., f-RBS braking. Power of the system can also be included in the calculations to determine the time of energy transfer. In the calculations, it is considered that two f-RBS systems are acting on both the front wheels so the total inertial load of the vehicle is distributed among two flywheels, i.e., the flywheel moment of inertia in the equation is equal to that of two flywheels.

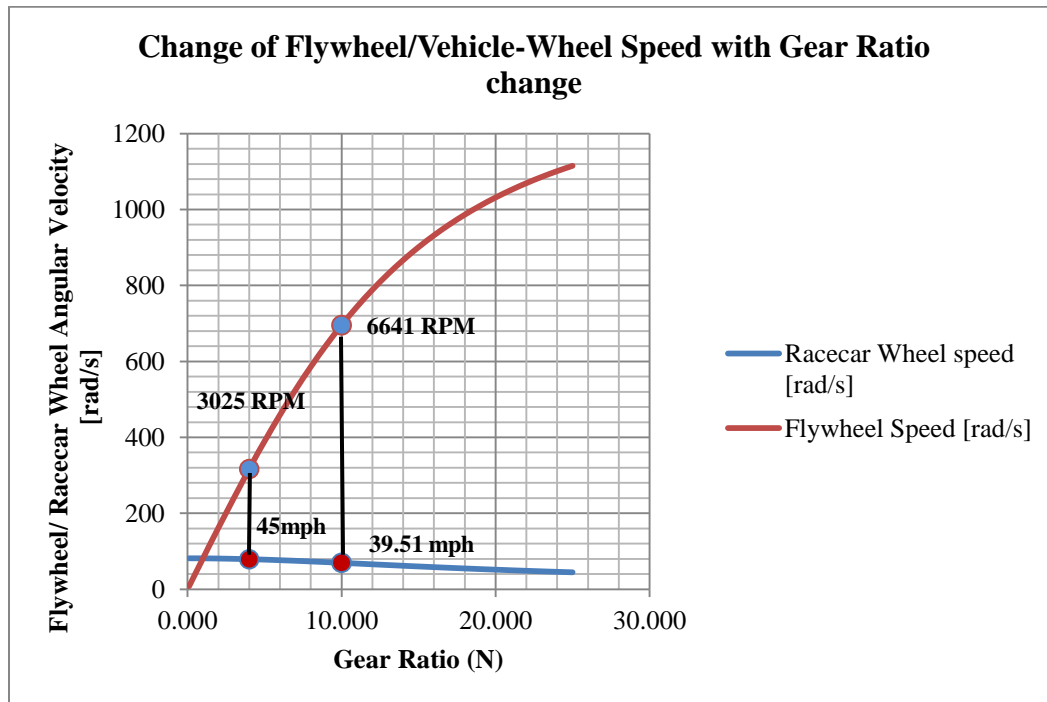


Figure 24: Energy transfer during a gear change between a flywheel and wheel

Also, the improvement in performance was found out using the equation (34) above. Using, Power of the system= 14 kW (7x2 kW), the minimum torque (moment of inertia times the angular acceleration

of the wheel) and hence the minimum force delivered by the f-RBS(in the range of operation) to the wheel is calculated and included in the performance calculations code described before (in the Mass vs. Performance section). It was observed there was a gain of a minimum of 0.4 s gain due to the use of f-RBS system. The gain in performance outperforms the loss due to the mass addition by at least 0.3 s.

TESTING, RESULTS AND DISCUSSION

7.0 TESTING

Any scientific theory suggested is developed and validated with help of mathematical models and experimental testing. Virtual testing is one computational method developed over the recent past which bridges the gap between these two methods. It is the process of testing any principle/product using Computer-aided tools by developing virtual prototypes and testing in conditions which can closely mimic actual conditions with close to realistic assumptions. It has the benefits of mathematical models as it too is a computational tool and hence is inexpensive, convenient and flexible in implementation with relatively less time and resources required. For most engineering problems, the real life physics can be simulated using Virtual testing, so most of the results of experimental testing can be generated with help of virtual testing. Due to its flexible and economical nature, it is a useful initial testing or supplemental tool to experimental testing. On this basis, it was chosen for this research. [47]

A Virtual Testing Rig model was developed using MSC ADAMS, a multibody dynamics and motion analysis software, in order to validate the principle of operation of the flywheel Regenerative Braking System (f-RBS). With the help of the Virtual test rig model, the kinematics of the f-RBS is to be studied and the mathematical model developed in the previous section validated.

7.1.0 About MSC ADAMS

MSC ADAMS (Acronym: Automatic Dynamic Analysis of Mechanical Systems) is a powerful software to analyze the dynamics of mechanical systems, by solving complex linear/non-linear dynamics, kinematics, statics and quasi-statics problems pertaining to the system. ADAMS can be used to create virtual prototypes of mechanical products and also for testing them on a virtual platform, in order to optimize the design and function of the product. It can also be used to generate accurate loads and forces in systems as an input for any Finite Element Analysis (FEA) code. MSC ADAMS has various modules to integrate the various technologies in a system like vibration analysis, control system design etc. and

some other modules like ADAMS/Car, ADAMS/Engine for specific application based analysis. The basic ADAMS modules for the modeling process are created around the philosophy of “Build-Test-Review-Improve” with each step of the process customizable to a large extent. [47]

7.1.1.0 ADAMS/View:

ADAMS/View is essentially the ‘Build’ Module of ADAMS, but is also used to submit simulation/test cases to ADAMS solver. It is the front-end GUI, which provides a 3D environment to build the mechanical models with using different ‘parts’, connecting parts using joints to constrain some degrees of freedom of individual parts relative to the system, apply different types of forces, torques, contacts and motions to parts and joints to study individual and system kinematics and dynamics. The virtual prototypes can also be tested, animated and optimized on ADAMS/View with the help of simulations, design studies, optimizations studies etc. [48]

7.1.2.0 ADAMS/Solver:

Solver is the numerical analysis application. Once a model is built, and inputs given, it is submitted to Solver to apply equations and automatically solve them. Program scripts can be written by users to control the simulation process externally. The user can also set the parameters for solving the equations of motion like the time step for integration, the type of integrators to use, the minimum error of approximation etc., for the solver externally. The results generated can be viewed on View with complete set of results data accessible at the Post Processor Module. [48]

7.2.0 f-RBS Virtual Test Rig - ADAMS Model

The Virtual test rig model was created in ADAMS/View as two rotating inertias connected to each other through a transmission, with one of them being the flywheel (part of f-RBS) and the other, the wheel, representing the vehicle mass and rotational inertia. The transmission consisted of the flywheel fixed gearing and the CVT and the through the control of this transmission, the energy exchange between

the two inertias, was controlled. The ADAMS Model is a scaled down model with the scaling of $\frac{1}{2}:1$ to the real model. This is in terms that the design of the f-RBS system is such that a full scale racecar requires two f-RBS systems at each front wheel for proper functionality and to distribute the total load of the vehicle. In this test rig, one f-RBS system is connected to half the vehicle's inertia to mimic the full scale.

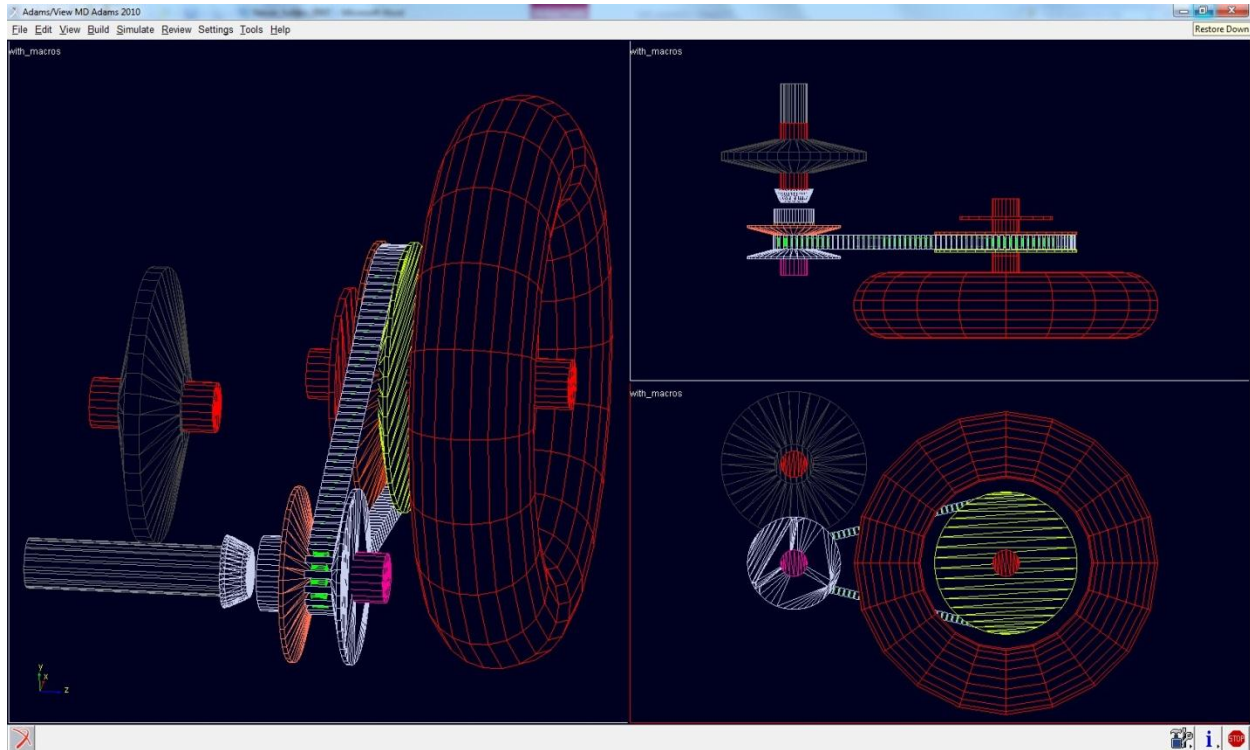


Figure 25: Virtual Test Rig – ADAMS Model in ADAMS/View

The flywheel model is imported from the SolidWorks design as a parasolid (.x_t format), and wheel is modeled in shape of a torus with the following dimensions and properties:

Table 9: Vehicle Equivalent Wheel ADAMS model properties

Wheel (torus) Dimensions and Properties	
Major Radius [mm]	180
Minor Radius [mm]	50
Mass [kg]	150
Moment of Inertia (about rotational axis) [kg-mm ²]	1.01E+07

The equivalent moment of inertia is calculated using equation (31). Both the flywheel and wheel are connected to the ground via a revolute joint allowing rotational motion with respect to the ground. The flywheel shaft was connected to a gear shaft via a 4:1 Gear Joint. This gear shaft and the CVT driveshaft engage and disengage with each other to connect the flywheel to the system through a simple cone clutch design. On activation the cone clutch allows for gradual engagement and disengagement between the flywheel system and the vehicle system.

7.2.1.0 CVT Modeling

A rubber belt VDP type CVT was chosen as the base template for the modeling. Due to its simple design, Comet CVT's specifications were used as the basis for dimensioning the CVT in ADAMS[44]. The CVT ratio range calculated from the design was 4.7:1 to 1:1 and the fixed gearing was 4:1 to give the test rig a theoretical operational range of 18.8:1 to 4:1.

Both driver and driven pulleys consist of two truncated conical pulley plates sitting on their respective shafts with one of the plates free to slide and other fixed with the shaft. Opposite plates of the pulleys have the freedom to slide to keep belt plane constant. Spring force is connected to both the moving plates to keep them in position and keep the tension in the belt.

Modeling the Rubber V-belt was the most complicated of tasks. It was modeled like a chain by creating a single V-belt part and linking many such parts together to form a belt. Since this was a repetitive process, ADAMS/View Macros was used to create small 'Macro' programs to perform the following functions to model the belt:

1. Establish design points as a trace of belt in space and all around the pulleys
2. Create steel links parts at the design points and import the rubber v-belt parts at each of these links.
3. Connect the each v-belt-links to next one via revolute joints.

4. Establish friction type contact forces between the belt parts and pulley plates and normal contact between individual pulley plates. Each of the v-belt-link part comes in contact with each of the pulley plates, so that's makes it 4 contacts forces for each part.

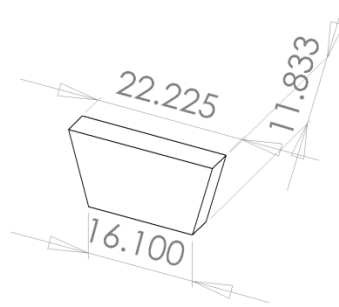


Figure 26: Drawing of V-Belt Part

The gear ratio of the system is controlled by assigning translational motion to the movable driver and driven pulley plate. The translational motion can be defined with help of functions or numerical values to define the displacement, velocity or acceleration of the pulley plate as a function of time. For initializing or giving a part an initial velocity or acceleration inn ADAMS/View, the motion of each parts can either defined by setting a motion at any of the joints or by giving initial velocity or displacement conditions to individual parts.

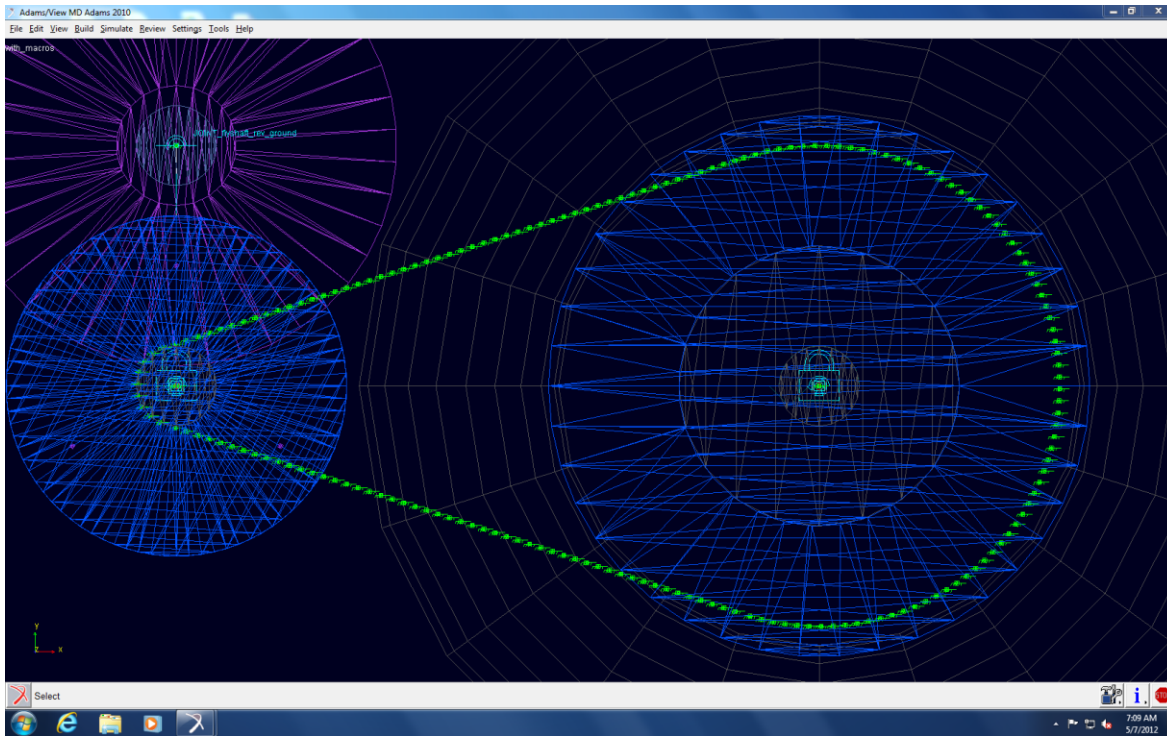


Figure 27: Macro establishing design points for belt model

7.3.0 Preliminary testing

Before virtual testing/simulation was performed, using Figure (18), the operational speed range of the flywheel and vehicle of the ADAMS Model were understood. Two preliminary tests were conducted, first to determine the actual gear ratio range of the CVT, second in order to establish the regenerative braking principle employed in this thesis.

7.3.1.0 Prelim Test I and Results

In order to determine the actual gear ratio range of the CVT and hence the transmission, the wheel was given an initial RPM and the CVT actuated through the full range (i.e., the pulley plate –both the driver and driven pulleys was displaced from zero position to 16.1 mm- the maximum separation between driver plates for highest gear ratio) under no load conditions i.e., the flywheel plus gearshaft system was disengaged from the system through the cone clutch. Once the conditions were set an ADAMS simulation was ran for the test to be performed with solver settings of Dynamics Solver and with a GSTIFF

integrator and I3 formulation (default and faster methods). From the results obtained, the maximum and minimum gear ratio achievable by the CVT and hence the transmission were noted. Also the curve of CVT ratio vs. pulley plate displacement was plotted and the equation derived to understand the relationship.

The maximum and minimum gear ratio of the CVT was observed to be 3.86:1 and 1.25:1 respectively, which worked out to total transmission ratio limits of 14.5 to 5:1. The CVT ratio vs. driver/driven pulley plate displacement plot equation was determined to be

$$n(x) = 3.8e^{-0.069x} \quad (35)$$

where, x : displacement of driver/driven pulley plate

$n(x)$: the gear ratio of CVT as a function of the displacement of driver/driven pulley plate

The same simulation solver settings are used in all tests below and hence wouldn't be mentioned again.

7.3.2.0 Prelim Test II and Results

The flywheel was initialized with an RPM of 3500 RPM and the wheel was given an initial velocity of 56.15 km/h (\approx 586 RPM), making the initial total gear ratio of 5.95 implying the CVT ratio was $5.95/4 = 1.4875$ initially. So, to match the flywheel and wheel speeds, the CVT driver and driven pulley plate were actuated (given translatory motion) for the ratio of the CVT to be 1.4875 (using the equation derived in the first test). Once the ratio was set, the cone-clutch was made to engage for both systems to connect, after which, for the energy transfer towards braking or charging the flywheel, the CVT ratio was made to switch to 3.01 (total transmission ratio of 12.02) by actuating the driver pulley plate. The whole process is performed by using a ADAMS/Solver Simulation script. The Simulation script contains instruction for the solver to perform the above mentioned tasks in a stepwise manner.

At the end of simulation, energy transfer was observed with the flywheel RPM of 5159 and the wheel velocity decreasing to 50.00 km/h. The following plot shows the exchange of energy between the wheel and the flywheel while regenerative braking, because as the wheel angular velocity decreases the flywheel angular velocity increases with time and gear change. This validates the operating principle behind the f-RBS concept.

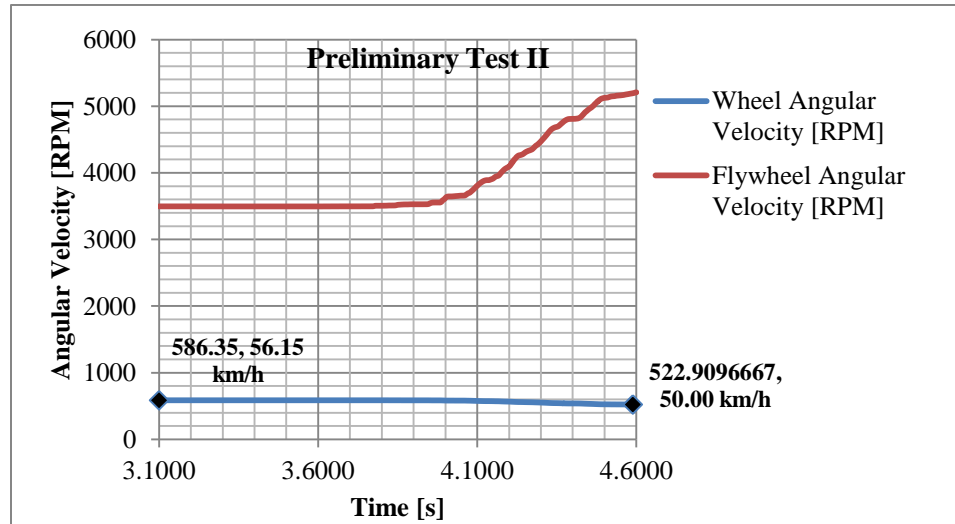


Figure 28: Plot for Preliminary Test -II

7.4.0 Drive Cycle Testing

In this phase, the f-RBS system was virtually simulated on the ADAMS Model for analyzing how useful the system is in actual drive conditions. For this purpose, a portion of test lap data from racecar DAQ (Data Acquisition) shown in figure (12) previously, is used in the simulation of the test rig.

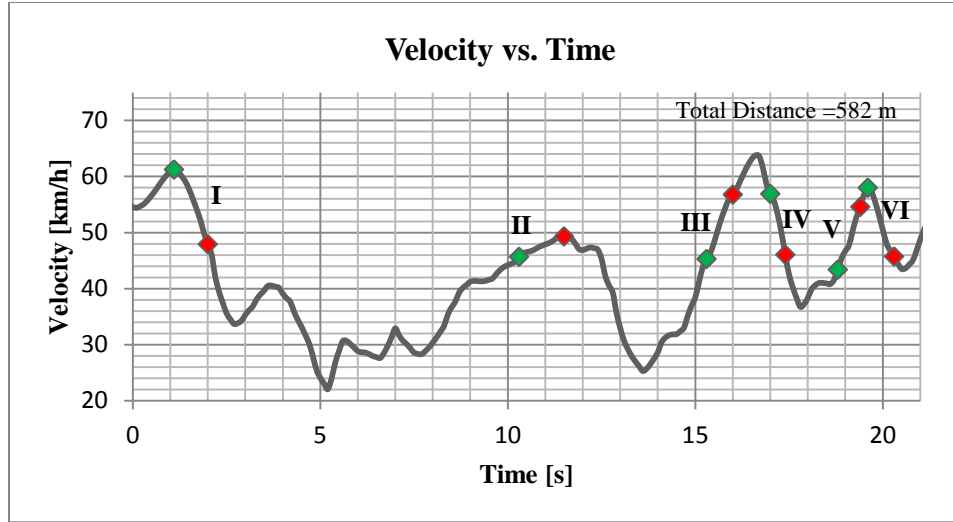


Figure 29: Lap Data Drive Cycle - With Cases to be simulated on the Rig

The portion is divided into six parts of braking and acceleration as shown below:

Table 10: Drive Cycle Test Cases

Case #	Initial Wheel Velocity	Initial Flywheel Velocity	Gear Ratio Initial (transmission)	Final Wheel Velocity	Final Gear ratio	Time [s]	Power
I	38 mph (\approx 61.155 km/h)	3500 RPM	5.48	30 mph (\approx 48.28 km/h)	14.47	1.214	7 kW
II	29 mph (\approx 46.67 km/h)	7300 RPM	14.36	31 mph (\approx 49.89 km/h)	12.13	0.27	7 kW
III	28 mph (\approx 45.06 km/h)	6600 RPM	14.02	35 mph (\approx 56.33 km/h)	5.06	0.99	7 kW
IV	36 mph (\approx 57.94 km/h)	3200 RPM	5.3	29 mph (\approx 46.67 km/h)	13.7	1.015	7 kW
V	27.5 mph (\approx 44.26 km/h)	6670 RPM	14.4	34 mph (\approx 54.72 km/h)	6.6	0.953	7 kW
VI	36 mph (\approx 57.94 km/h)	3507 RPM	5.8	28.5 mph (\approx 45.87 km/h)	14.5	1.08	7 kW

Each of the above cases is ran as individual simulations on the ADAMS Virtual test rig model and the results are plotted with the velocity-time graph of the lap data (Figure (29)). The mathematical model predictions are also plotted on same plot for comparison of the results. Since it is an open loop test, the flywheel is initialized at the start of Case I with the minimum flywheel RPM of 3500 RPM. The minimum flywheel RPM is the angular speed of the flywheel below which the f-RBS system cannot be

utilized and this is determined using Figure (18) and transmission gear ratio range calculated in the preliminary test I.

7.5.0 Results and Discussion

The f-RBS was simulated using ADAMS Model through all the cases of the lap data drive cycle. The results for each case were pieced together and the necessary extrapolation was done to show the total usage of the flywheel start to end of the portion of lap data. The following plot was obtained based off Figure (29):

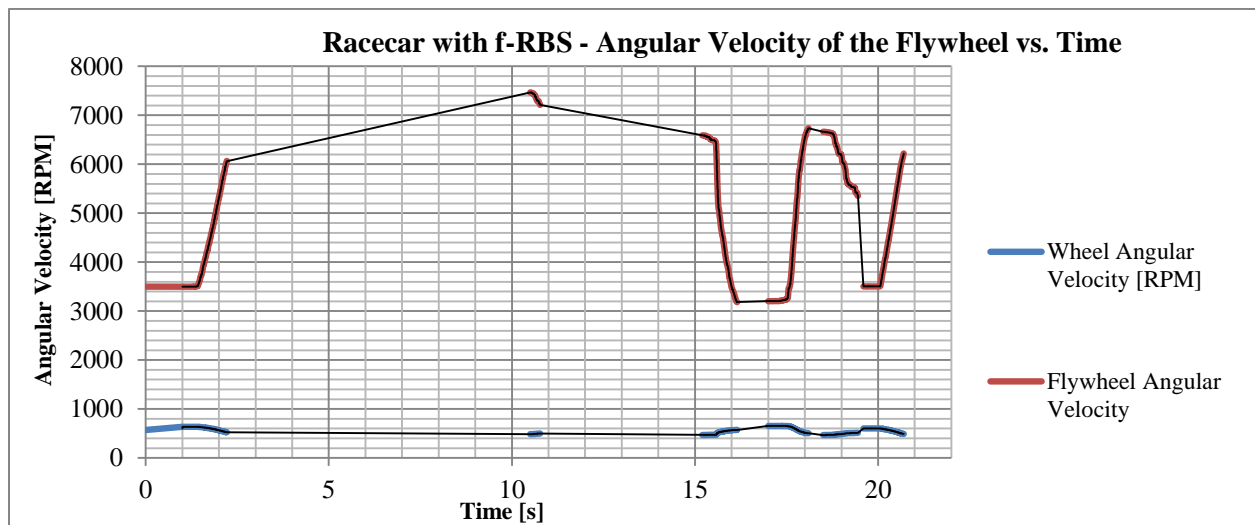


Figure 30: Results of f-RBS simulated through part of lap as shown in Figure (29)

This plot shows the angular velocities variation of flywheel when the wheel velocity is forced to decrease (regen braking) or increases (Accel –boost) by the desirable amounts given in the Table 10. The flywheel starts at its operating minimum of 3500 RPM and reaches 6100 RPM by the end of the lap portion. The results also prove that for a half-scale racecar with one f-RBS system of 7kW power, considerable vehicle velocity changes are possible and hence significant regen-braking and Accel boost can be achieved. For a full scale racecar, two f-RBS system need to be used for distributing the load and achieve the same results. With a wider transmission ratio range, a wider range of operational velocities for both flywheel and vehicle can be achieved.

Individual case by case plots are shown below for comparing flywheel RPM results from the virtual test rig and the mathematical model. It can be seen from all the plots that there is a small error in the test rig result on comparison with the mathematical model results. This error can be attributed to the losses in energy/velocities while engaging/disengaging through the cone clutch. If a more efficient clutching system was used, like an electric clutch these losses can be overcome easily.

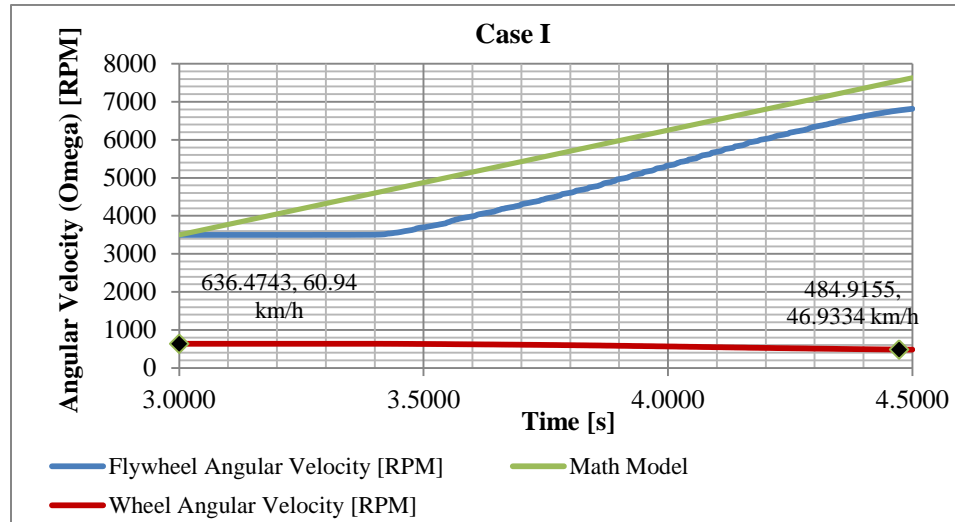


Figure 31: Plot of Case I – Comparison of Math model results and ADAMS Model results

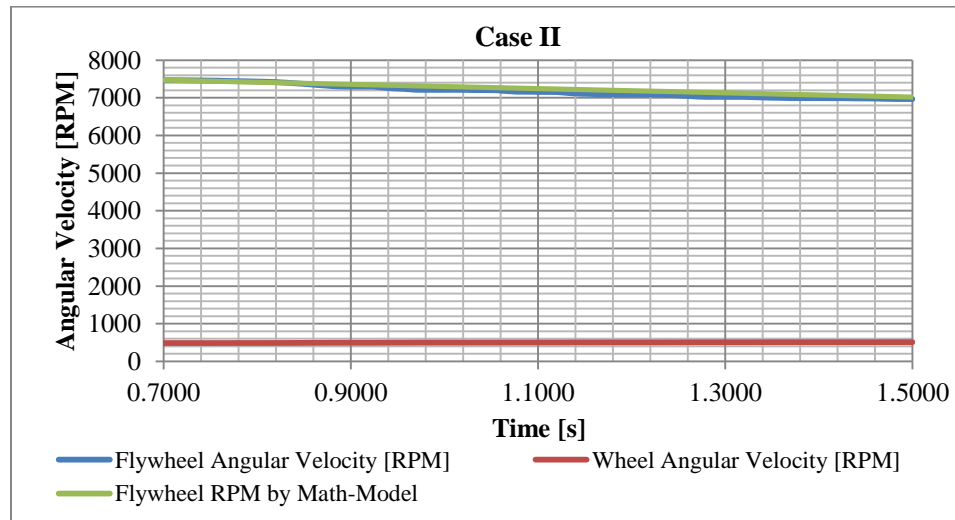


Figure 32: Plot of Case II – Comparison of Math model results and ADAMS Model results

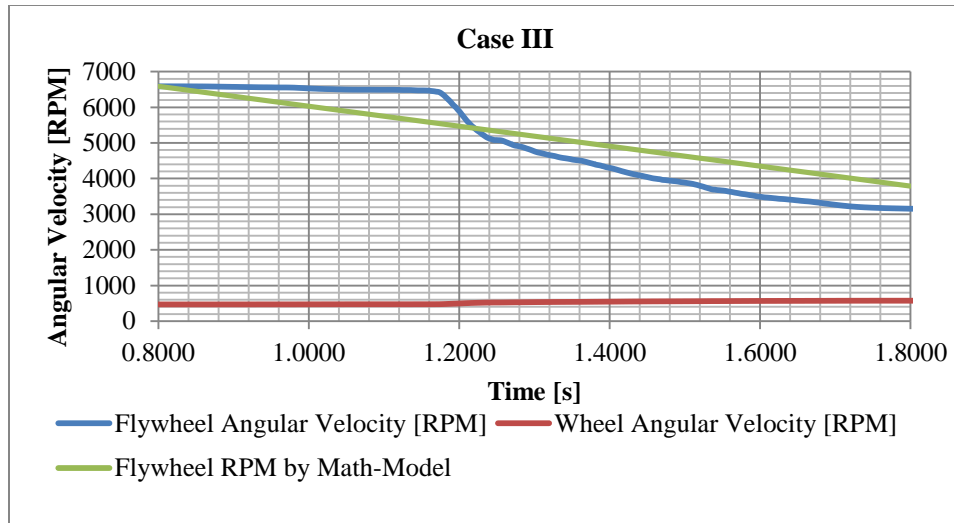


Figure 33: Plot of Case III – Comparison of Math model results and ADAMS Model results

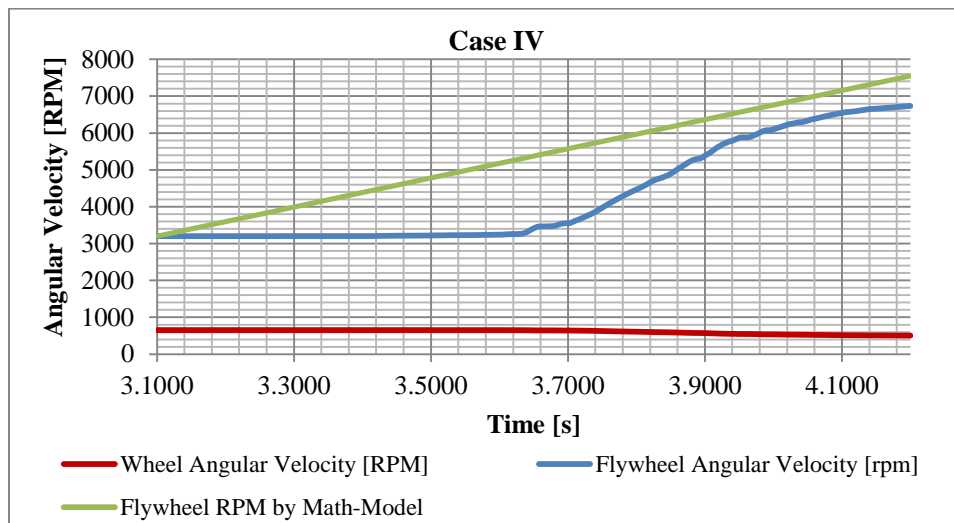


Figure 34: Plot of Case IV – Comparison of Math model results and ADAMS Model results

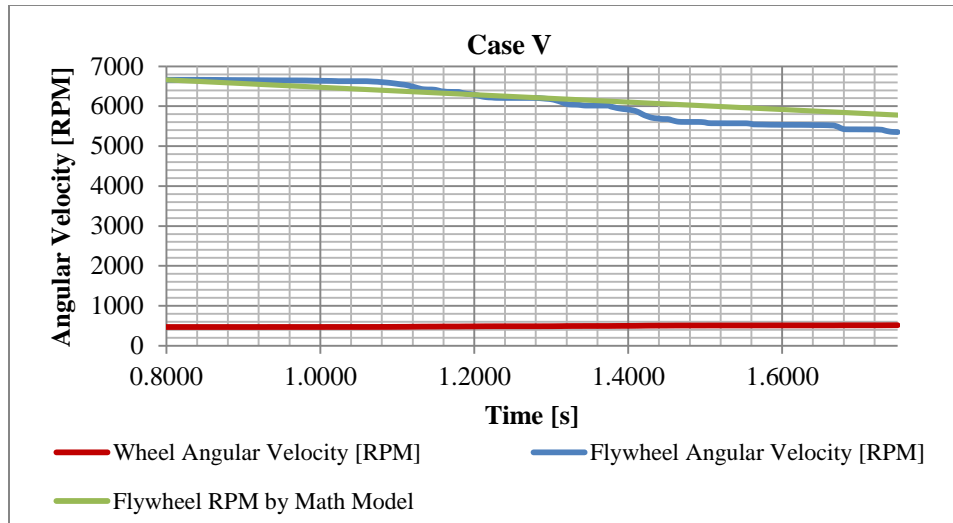


Figure 35: Plot of Case V – Comparison of Math model results and ADAMS Model results

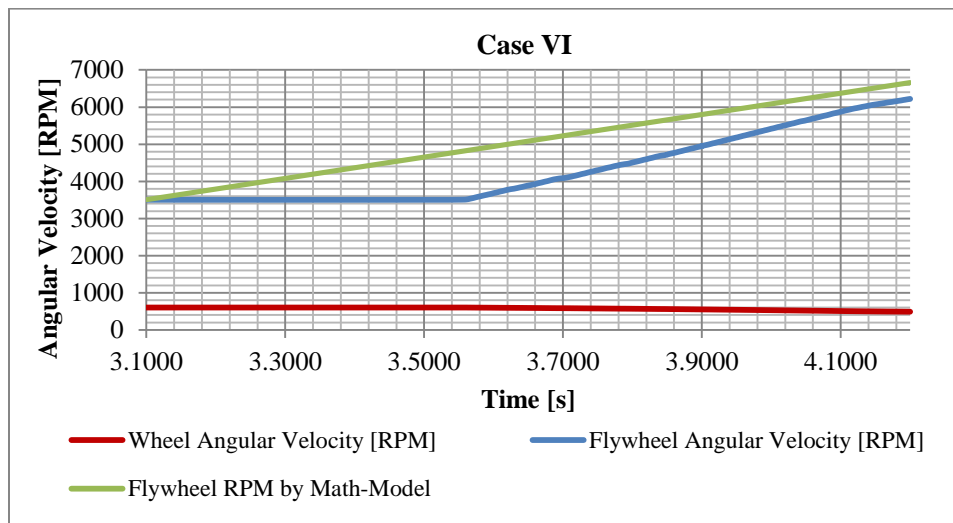


Figure 36: Plot of Case VI – Comparison of Math model results and ADAMS Model results

7.6.0 Applications of the ADAMS Virtual Test Rig Model

The Virtual Test Rig model has been successfully utilized to validate mathematical data and initial testing of the f-RBS system. Furthermore it can be used for various testing purposes like:

1. Run simulation of f-RBS utilization in a full drive cycle to study the energy transfer, to compare results with the mathematical model and to observe the performance gain/loss due to the system.

2. With the addition of road-tire contacts using ADAMS/Car, road losses can also be embedded into the model as force functions.
3. Calculate torque and power characteristics of the system
4. Understand the basic controllability of the system, and embed control systems into to the model and further understand that function.
5. Calculation of actuation force required for the pulley plate and actuation control functions too.
6. Fuel Efficiency calculation by calculation how much energy needs are being satisfied during a drive cycle /lap.
7. Testing using different belt materials and different CVTs.

8.0 CLOSURE

8.1.0 Summary and Conclusion

The design principles for a f-RBS concept for a Formula SAE type racecar have been presented in this thesis. In addition, modeling of a Virtual Test rig for the f-RBS testing using MSC ADAMS and initial testing results of the f-RBS on the Virtual test rig have also been presented.

For a RBS solution for the racecar, a comprehensive literature review was performed and presented on RBS technologies and mechanical RBS technologies. As a result of the research, a flywheel based mechanical regenerative braking system (f-RBS) was conceptualized for a Formula SAE type racecar application. Utilizing the racecar data, the design principles for the f-RBS and its components namely – transmission and flywheel were established. Operation and integration principles of such a system with the racecar were realized and extended to the current application.

A mathematical model was derived to explain the operating principle of regenerative braking and acceleration using the f-RBS system. A virtual test rig model was also created using MSC ADAMS to validate the mathematical model results and perform initial testing as an alternative to experimental testing. Virtual test rig ADAMS Model results successfully proved the validity of the test rig as a preliminary, cost effective and flexible alternative to experimental testing. The potential of the test rig for future testing was also realized. On the basis of the test rig results and mathematical model, the operating principle of the f-RBS was validated. Also, the results proved that the system can be a useful tool for improving the performance of the racecar and recycling wasteful braking energy.

8.2.0 Future Scope

The following areas need to be addressed in future work to complete the design and testing of the f-RBS concept before implementation:

1. Incorporation of the actuator mechanism in the CVTs for external control of the transmission if necessary.
2. Vacuum and Containment Design for the flywheel for reduction of windage losses.
3. Bearing and clutch selection to reduce friction losses.
4. Incorporation of Carbon Fiber or other composite material for high speed flywheel design instead of metal low speed with the appropriate transmission-CVT for improving safety from shattering of flywheels.
5. Rotational Dynamics analysis from the vehicle standpoint as to how inclusion of two rotating/counter-rotating flywheels from two f-RBS systems can affect the dynamics of the moving racecar.
6. A more comprehensive failure and fatigue analysis for safety of the system
7. Control system design for the f-RBS operation in conjunction with the vehicle.
8. Cooling system design if required.
9. Physical/Experimental testing on a Rig and vehicle.

On the front of the virtual test rig, it can be developed furthermore to incorporate all the possible applications mentioned above in 7.6.0. The current model needs to be run manually and individually for every acceleration and deceleration event. Instead, it can be modified to run a complete cycle with a few simple inputs with the help Solver scripts, sensors, Macros and GUI developing tools. Also road, bearing and clutch losses need to be incorporated for a complete dynamic analysis of the f-RBS system.

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APPENDIX

